Bergen Open 2018
Solution Slides

November 10, 2018

UNIVERSITY OF BERGEN

The Jury

- Olav Røthe Bakken
- Petter Andre Dahl Elvevoll
- Øyvind Stette Haarberg
- Torstein Strømme
- Birk Tjelmeland
- Erik Tjøswold
- Amar Topalovic
- Magnus Øian

License to Launch

- Problem summary: Find first occurrence of smallest value in list of numbers.
- Algorithm:
  - Check each value in the provided order
  - Keep a running minimum value and remember its index
  - Print the remembered index when loop is finished
- One-liner (python3):

```python
print(min((v, i) for i, v in enumerate(int(x) for x in input().split()))[1] if input() else "")
```

- Runtime: *O(n)*

Author: Birk Tjelmeland **First solved**: 00:07 **Solved by**: 33 teams

Fishmongers

- ➢ Problem summary: Sell fish to make as much monies as possible.
- ➢ Algorithm:
  - Sort your fish w.r.t weight
  - Sort fishmongers w.r.t price
  - Sell your biggest fish to the buyer who is willing to pay the most, until either no more fish or no more buyers
- ➢ Runtime: $O(n \log n)$

Author: Torstein Strømme and Øyvind Stette Haarberg **First solved**: 00:17 **Solved by**: 12 teams

Awkward Party

- Problem summary: Given a list of integers, find the shortest distance between any pair of equal integers.
- Algorithm:
  - Maintain a dictionary which maps each integer to its previously seen position.
  - Keep track of the shortest distance d.
  - For each integer in the list:
    - If integer is encountered previously, check if difference between previous and current position is less than d and update accordingly.
    - Update previous position of the given integer.
- Runtime: $O(n)$

Author: Magnus Øian and Torstein Strømme **First solved**: 00:23 **Solved by**: 18 teams

Joint Attack

- Problem summary: Given a number x as a continued fraction, output x as a reduced fraction.
- Algorithm:
  - Keep track of numerator (`num`) and denominator (`den`) for each layer.
  - Starting at the bottom-most layer:
    - Find denominator and numerator of integer plus fraction.
    - Reciprocate (switch `num` and `den`) to eliminate a layer.
    - Simplify fraction.
    - Repeat until only one fraction left.
  - Print remaining fraction (`den + "/" + num`).

$$x = x_o + \frac{1}{x_1 + \frac{1}{x_2}}$$

↓

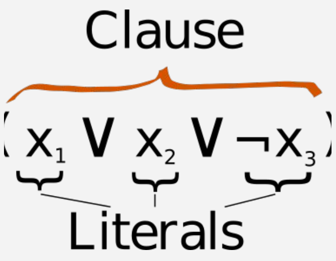
$$x = x_0 + \frac{x_2}{x_1 x_2 + 1}$$

↓

$$x = \frac{x_0 + x_2 + x_0 x_1 x_2}{x_1 x_2 + 1}$$

Author: Olav Røthe Bakken **First solved**: 00:27 **Solved by**: 15 teams

Counting Clauses



- Problem summary: Determine whether a given SAT formulae has eight clauses or more.
- Algorithm:
  - Read first number of input
  - If that number is ≥ 8, print "satisfactory"
  - Otherwise print "unsatisfactory"
- One-liner (python3):

```
print("satisfactory" if int(input().split()[0]) >= 8 else "unsatisfactory")
```

Author: Øyvind Stette Haarberg **First solved**: 00:36 **Solved by**: 26 teams

Keyboards in Concert

- Problem summary: Play a tune with instruments, each with limited access to notes. Switch instrument as little as possible.
- Algorithm:
  - Observe that we want to start with the instrument which gets us the farthest
  - Don't need to compute this; simply maintain list of valid instruments after each note
  - When no more valid instruments: Increment a counter and reset group of valid instruments
- Runtime: $O(mn + nk)$

Author: Olav Røthe Bakken **First solved**: 01:18 **Solved by**: 7 teams

Backpack Buddies

- Problem summary: A race between two players in a weighted graph; Mr. Day is required to end each 12 hour increment at a vertex, whereas Dr. Knight need not.
- Algorithm:
  - Run normal Dijkstra to determine walking time required for Dr. Knight. Compute time she spent resting.
  - Run a special Dijkstra for Mr. Day where he ends every day at a vertex
    - Assume Mr. Day arrives at vertex u at day d and hour h
    - Assume there is an edge from u to neighbour v which takes w hours to traverse. Then:
      - If $h + w \leq 12$, it is possible to arrive at v at day d and hour $h + w$
      - Otherwise, it is possible to arrive at v at day $d + 1$ and hour w
- Runtime: $O(m \log n)$

Author: Petter Elvevoll and Torstein Strømme **First solved**: 02:07 **Solved by**: 1 team

Hidden Words

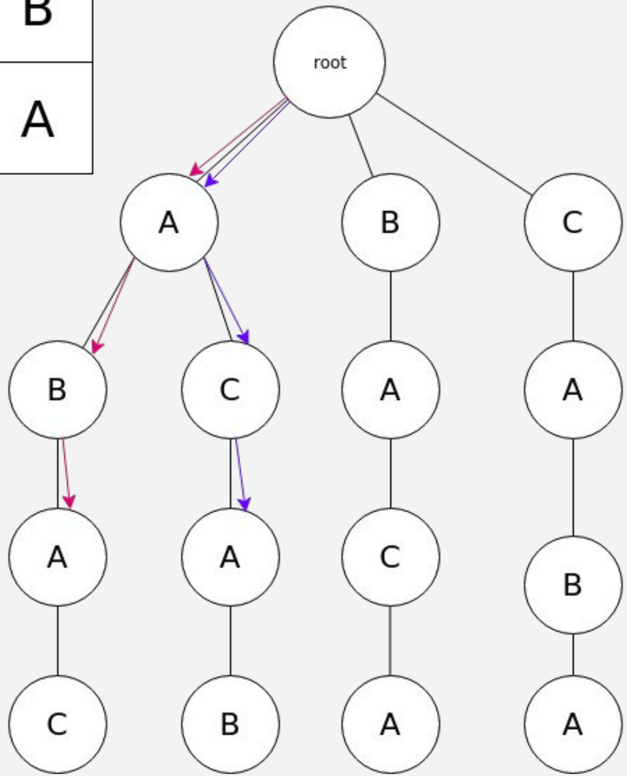
S	N	K	O
V	R	E	R
I	D	I	N

- Problem summary: Given a grid of letters and a list of words, count the number of words in the list that occur in the grid
- Algorithm:
  - First construct every possible word in the grid:
    - Start from every cell: Run DFS with max “depth” 10 that unmarks cell as visited after visiting neighbors
    - While in the DFS, construct a trie from the found words
    - Observe: (significantly) less than $\sum_{i\in 0..9} h\cdot w\cdot 4\cdot 3^{i-1} \approx 3\cdot 10^6$ such words.
  - For every word in the list, increment a counter if it exists in the trie
- Runtime: $\sim 4\cdot 3^8 hw + 10n$

Author: Amar Topalovic **First solved**: 02:43 **Solved by**: 3 teams

Hidden Words

A	B
C	A



Words in Trie:
A
AB
ABA
ABAC
AC
ACA
ACAB
B
BA
BAC
BACA
C
CA
CAB
CABA

Author: Amar Topalovic **First solved**: 02:43 **Solved by**: 3 teams

Expecting Rain

- Problem summary: Get to the bus as dry as possible within time limit.
- Algorithm:
  - Define **dp[i][j]** as the minimum amount of rain you can expect being at position *i* at time *j*
  - Want to find **dp[d][t]**
  - Base case observations:
    - **dp[0][j]** = 0 (since there is always roof at distance 0 from home)
    - **dp[i][0]** = ∞ (unless *i* = 0) (since we must start from home)
  - Observe: Can assume all waiting happens at roof *endpoints* (notable exception: the bus stop itself, if roofed)
  - Recurrence:
    - If roof endpoint at position *i*, then **dp[i][j]** = min(**dp[i][j-1]**, **dp[i-1][j-1]**)
    - If no roof at position i, then **dp[i][j]** = **dp[i-1][j-1]** + expected rain at that time interval

Author: Øyvind Stette Haarberg and Torstein Strømme **First solved**: N/A **Solved by**: 0 teams

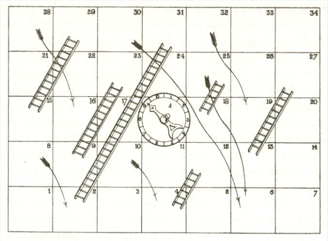
Expecting Rain

- Problem summary: Get to the bus as dry as possible within time limit.
- Algorithm:
  - Preprocessing: For each time unit, calculate the expected amount of rain using prefix sum
  - Calculate recurrence using dp table or memoization
  - Be careful with edge cases

- Runtime: $O(c + dt)$
  - d = distance to the bus stop, t = time until the bus leaves, c = number of clouds

Author: Øyvind Stette Haarberg and Torstein Strømme **First solved**: N/A **Solved by**: 0 teams

Dice and Ladders

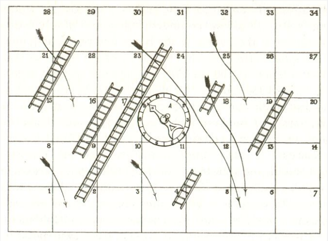


- Problem summary: Given a board of ladder game, find the smallest number *x*, such that after *x* dice rolls you win the game with probability at least *p*
- Background:
  - Define *P* (*x*) to be true if you can win the game with probability at least *p* with x dice rolls, false otherwise
  - *P* has the property required for binary search: if *P* (*x*) then also *P* (*x*+1)

  - Define $\mathbf{M_i[s, t]}$ as the probability of moving from cell *s*, to cell *t* in *i* rounds
  - Want to find smallest *x* such that $\mathbf{M_x[1,}\ \boldsymbol{c \cdot r}\mathbf{]} \geq p$
  - Observe: Matrix $\mathbf{M_1}$ can be constructed by examining input
  - Observe: $\mathbf{M_i[s, t] = \Sigma_k(M_{i-1}[s, k] \cdot M_1[k, t])}$, which implies $\mathbf{M_i} = \mathbf{M_{i-1}} \times \mathbf{M_1}$ and thus $\mathbf{M_i} = (\mathbf{M_1})^{\mathbf{i}}$

Author: Birk Tjelmeland **First solved**: N/A **Solved by**: 0 teams

Dice and Ladders

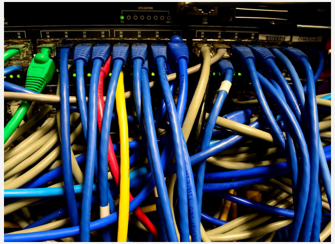


- **Algorithm:**
  - Construct matrix $\mathbf{M}_1$
  - Binary search on the number of dice rolls, x:
    - Compute $\mathbf{M}_x = (\mathbf{M}_1)^x$
    - If $\mathbf{M}_\mathbf{x}[\mathbf{1}, \boldsymbol{c \cdot r}] \geq p$ try smaller x, else try larger x

- **Runtime complexity:**
  - TLE with naive matrix exponentiation
  - AC with fast matrix exponentiation: $a^b = \left(a^{\frac{b}{2}}\right)^2$
  - Final complexity: $O((c \cdot r)^3 \log^2 x)$
  - Can be done more cleverly in $O((c \cdot r)^3 \log x)$

Author: Birk Tjelmeland **First solved**: N/A **Solved by**: 0 teams

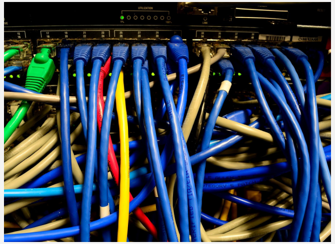
ISP Merger



- Problem summary: make a graph connected with at most k edge additions or deletions, without violating degree constraints
- Structural insights:
  - When we connect two components we add an edge between one vertex with open connection sockets in component 1, and a vertex with open connection sockets in components 2.
  - If we have a component with no free connection spots, we must delete an edge to obtain free spots to connect to other components.
  - We don't care about the size of each connected component or how a connected component is connected except for two details: the number of free connection spots, and the number of removable edges (we can calculate this number by seeing how many more edges than a tree this component has; i.e. since trees have n-1 edges, a component with m edges will have m-(n-1) removable edges.)

Author: Øyvind Stette Haarberg **First solved**: N/A **Solved by**: 0 teams

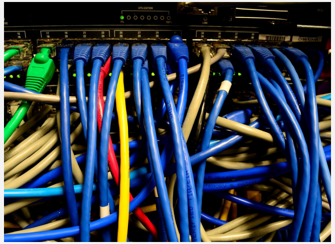
ISP Merger



- Structural insights (cont'd):
  - We only care about the free spots and number removable edges for all connected components
  - We want to connect the components with the most free spots first to minimize deletions
  - Trees with no free connection spots and trees with 1 free connection spot cause difficulties
  - We can't connect a tree with no free connection spots to another component without violating a degree constraint. We must therefore split all such trees, obtaining two trees with one free connection spot
  - When we connect two trees with one free connection spot together we will obtain a tree with no connection spots. We only want to do this as a last step, as the resulting component can't be connected further.

Author: Øyvind Stette Haarberg **First solved**: N/A **Solved by:** 0 teams

ISP Merger

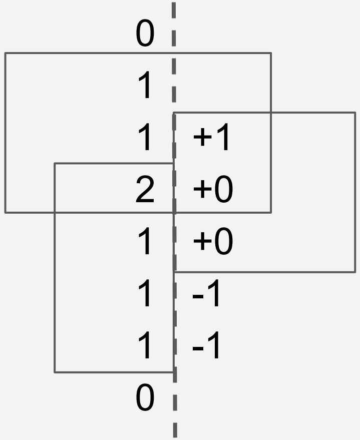


- Algorithm:
  - Find the number of free connection spots and removable edges for each component (find only 1 → “yes”)
  - Split up all components which are trees with no connection spots
  - While we have more than one component and $k \geq 0$:
    - Take the two components with the most free connection spots (exception: trees with one connection spot are sorted last)
    - Make sure they have at least free connection spot (delete a non-bridge edge if not -- if no such edge and no free spot output “no”)
    - Connect them together
  - Make sure to update k for every edit
- Runtime (with a priority queue for ordering components): $O(|V| \log |V| + |E|)$

Author: Øyvind Stette Haarberg **First solved**: N/A **Solved by**: 0 teams

Gameworld Tornado

- ➢ Problem summary: Compute area of rectangles
- ➢ Algorithm
  - Convert rectangles into events
    - Event consists of an *x* coordinate, and a segment (two *y* coordinates) and a delta value (+1 or -1)
  - Sort events by *x*
  - For each event
    - Add (x - lastx) · "score" of segment tree root to total area
    - Add event delta to segment tree within event delta
    - Update lastx
  - Output total area



Author: Magnus Øian and Olav Røthe Bakken **First solved**: N/A **Solved by**: 0 teams

Gameworld Tornado

- Segment tree
  - Each node contains a value and score
  - Value is number of rectangles contained within the segment
  - Score is the length covered within the segment
  - If value is positive then score is the size of the segment - else score is the sum of the scores of the children
- Runtime: $O(n \log n)$

Author: Magnus Øian and Olav Røthe Bakken **First solved**: N/A **Solved by**: 0 teams

Statistics

- Number of teams: 33
- Number of participants: 77
- Number of submissions: 463
- Number of accepted submissions: 115
- First accepted submission: 00:07:25 (License to Launch)
- Last accepted submission: 04:58:23 (Counting Clauses)
- Number of commits to problem repository: 489

Copyright notes