NCPC 2018
Presentation of solutions

The Jury

2018-10-06

NCPC 2018 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Andreas Björklund (Lund University)
- Markus Dregi (Equinor/Webstep)
- Bjarki Ágúst Guðmundsson (Syndis)
- Antti Laaksonen (CSES)
- Jimmy Mårdell (Spotify)
- Lukáš Poláček (Google)
- Torstein Strømme (University of Bergen)
- Pehr Söderman (Kattis)
- Jon Marius Venstad (Oath)

B — Baby Bites

Problem

Check that input list is $1, 2, \ldots, n$ except that some elements may be replaced by “mumble”.

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Obligatory Prolog Solution

```
solve(["mumble"|Tail], Pos) :-
    NewPos is Pos+1,
    solve(Tail, NewPos).
solve([Head|Tail], Pos) :-
    number_string(Pos, Head),
    NewPos is Pos+1,
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solve([], _) :- write("makes sense").
solve(_, _) :- write("something is fishy").
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Statistics: 453 submissions, 225 accepted, first after 00:02

C — Code Cleanups

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Given list of days of dirty pushes, each increasing dirtiness by 1 per day, calculate how many cleanups are needed to keep dirtiness below 20 at all times.

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2. Whenever dirtiness reaches ≥ 20, we needed a cleanup the evening before and reset everything

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Statistics: 669 submissions, 198 accepted, first after 00:11

H — House Lawn

Problem

Given specs for a bunch of lawnmowers, find cheapest ones with sufficiently high capacity for given lawn size.

Solution

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Solution

1. Mower with cutting rate c, cutting time t, recharge time r cuts on average $10080ct/(t+r)$ square meters per week

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2. Among those where this is at least lawn size, print names of cheapest mowers.

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Statistics: 842 submissions, 155 accepted, first after 00:25

I — Intergalactic Bidding

Problem

Given sequence of positive integers $a_1, \ldots, a_n$ such that $a_i \geq 2a_{i-1}$, find subset that sums to t.

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Given sequence of positive integers $a_1, \ldots, a_n$ such that $a_i \geq 2a_{i-1}$, find subset that sums to t.

Solution

1. Doubling property $\Rightarrow$ sequence *superincreasing*: $a_i > \sum_{j=1}^{i-1} a_j$

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Statistics: 328 submissions, 91 accepted, first after 00:24

J — Jumbled String

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Create a bit string that has given numbers of subsequences “00”, “01”, “10” and “11”, or report that this is not possible.

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   - if number of "00" is 0, two solutions $x = 0$ and $x = 1$
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   (you can also handle small cases using brute force).

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Statistics: 359 submissions, 38 accepted, first after 00:22

E — Explosion Exploit

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Given healths of your own and your opponent's minions in a card game, what is probability that all opponent's minions die after dealing d damage one at a time randomly to living minions?

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   Number of possible states is reduced to $\binom{5+7-1}{7-1}^2 = 213\,444$

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Statistics: 144 submissions, 41 accepted, first after 00:34

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2. We see that answer only depends on n and k, not on structure of T and get recurrence

$$f(n,k) = k \cdot f(n-1, k-1) + (k-1) \cdot f(n-1, k)$$

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3. Compute in your favorite way in $O(nk)$ time

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Given tree T on n vertices, how many k-colorings does it have that use all k colors?

Solution 2 [Inclusion-Exclusion]

1. Number of c-colorings (not necessarily using all c colors) is $c(c-1)^{n-1}$: root can have any color and as we go down the tree each node has $c-1$ choices

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2. By principle of inclusion-exclusion, answer is

$$f(n,k) = \sum_{c=1}^{k} (-1)^{k-c} \binom{k}{c} c(c-1)^{n-1}$$

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Statistics: 123 submissions, 33 accepted, first after 00:30

D — Delivery Delays

Problem

Given list of orders made and when they are ready to be delivered from origin to destination, what is smallest possible maximum delay to deliver them in first-come-first-served order?

Solution

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1. Find shortest distances between all pairs of nodes in the graph (using Dijkstra's algorithm for each vertex)

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Solution

1. Find shortest distances between all pairs of nodes in the graph (using Dijkstra's algorithm for each vertex)
2. Binary search for answer D
3. To check if delay D possible, let $L_D(i)$ be latest possible start time for delivering orders $i, i+1, \ldots, k$ with max delay D

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4. Compute $L_D(i)$: guess how many orders j to bring together with i, simulate delivering them, then use value of $L_D(i+j)$

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5. Time complexity is $O(nm \log m + k^2 \log D_{\max})$.

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Statistics: 40 submissions, 10 accepted, first after 01:25

A — Altruistic Amphibians

Problem

Given leap capacities, weights, and heights of a set of frogs, decide how many frogs can escape a pit of given depth d if they build piles of frogs to elevate each other. No frog can carry its own weight.

Solution

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Solution

1. A frog can only help lighter ones, so can assume the frogs leave pit in order of increasing weight.

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2. Maintain array $H[w]$ = height of highest frog pile that can carry a weight of w (for w up to max weight).

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2. Maintain array $H[w]$ = height of highest frog pile that can carry a weight of w (for w up to max weight).
3. For each frog (l_i, w_i, h_i) by *decreasing weight* (time reversal):
   1. Frog escapes if $l_i + H[w_i] > d$
   2. Update $H[w] = \max(H[w], h_i + H[w_i + w])$ for $1 \leq w \leq w_i - 1$

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Statistics: 55 submissions, 1 accepted, first after 04:29

G — Game Scheduling

Problem

Given m teams with n players per team, construct a round based schedule so all players play against all players from all other teams, such that each player has at most one bye (free round).

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Solution 1 [Explicit construction]

1. The $n = 1$ case: classic round robin scheme

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   3. n and m odd: use one last round to collect remaining games.

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2. A schedule using $d = n(m-1)+1$ days is an *edge coloring* of the graph with d colors.
3. **Vizing's Theorem**: *every graph has an edge coloring using $\Delta + 1$ colors where Δ is max degree.* Exactly what we need.

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Given m teams with n players per team, construct a round based schedule so all players play against all players from all other teams, such that each player has at most one bye (free round).

Solution 2 [General solution]

1. Construct graph with $m \cdot n$ nodes representing all players, with edges between players from different teams.
2. A schedule using $d = n(m-1)+1$ days is an *edge coloring* of the graph with d colors.
3. **Vizing's Theorem**: *every graph has an edge coloring using $\Delta + 1$ colors where Δ is max degree.* Exactly what we need.
4. Use efficient algorithm for finding a $(\Delta + 1)$-edge coloring (Misra-Gries). Naive implementation sufficiently fast.

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Statistics: 10 submissions, 0 accepted

F — Firing the Phaser

Problem

Given set of axis-aligned rectangles, find max number of rectangles that can be intersected by a straight line segment of length ℓ.

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1. Idea: given the line on which the optimal segment lies, we get a relatively easy one-dimensional problem about intervals.
2. So we just have to find a small candidate set of lines (= pairs of points) to try.

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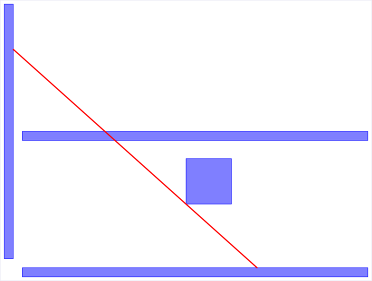

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Statistics: 19 submissions, 0 accepted

Random statistics

232 submitting teams

3149 total number of submissions (792 accepted)

6 programming languages used by teams

Ordered by popularity: Python 2/3 (1400), Java (892), C++ (740), C# (105), C (6), Haskell (6)

(Top 3 languages are in reverse order from the "usual" one! Python, Java and C# increased in popularity, all other languages decreased.)

381 number of lines of code used in total by the shortest **jury** solutions to solve the entire problem set. (Much smaller than usual.)

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Each university sends up to two teams to NWERC to fight for spot in World Finals (April 2019 in Porto, Portugal)