

# **BAPC 2022**

Solutions presentation

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October 22, 2022

## E: Equalising Audio

Problem Author: Abe Wits

- **Problem:** Normalise the amplitudes  $a_1, \dots, a_n$  to a perceived loudness of  $\frac{1}{n} \sum_{i=1}^n a_i^2 = x$ .
- **Solution:** First, compute the current perceived loudness  $x' = \frac{1}{n} \sum_{i=1}^n a_i^2$ .  
Then, output  $\sqrt{x/x'} \cdot a_1, \dots, \sqrt{x/x'} \cdot a_n$  with sufficient precision.

- Verification:

$$\frac{1}{n} \sum_{i=1}^n (\sqrt{x/x'} a_i)^2 = \frac{x}{x'} \cdot \frac{1}{n} \sum_{i=1}^n a_i^2 = \frac{x}{x'} \cdot x' = x.$$

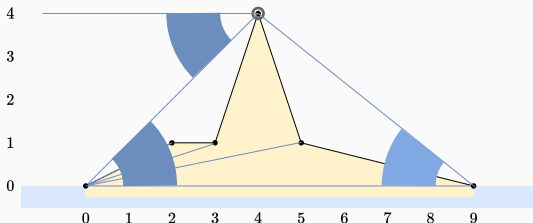
- Exception: if  $x' = 0$ , return  $0, \dots, 0$ .
  - And apparently, we forgot to test the case  $-1 \ 1$ , where the sum is 0
- Pitfall: multiplying two ints causes integer overflow, use long instead!

Statistics: 127 submissions, 50 accepted, 6 unknown

## B: Bellevue

Problem Author: Ragnar Groot Koerkamp

- **Problem:** Given the profile of an island, find the point with the largest viewing angle of the sea.
- **Observation:** If point  $P$  is blocking the view of the edge of the island from point  $Q$ , you can see more sea in  $P$  than  $Q$ .
- The answer is always an angle from the start/end of the island to another point.
- **Solution:** take the maximum angle around the start/end.



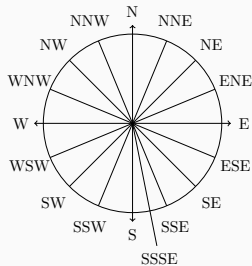
- **Alternative solution:** compute the convex hull and iterate over it.

Statistics: 102 submissions, 41 accepted, 18 unknown

# F: Failing Flagship

Problem Author: Ruben Brokkelkamp

- **Problem:** Compute the minimum angle in degrees between two wind directions.
- **Solution:**
  - Convert both wind directions into degrees  $d_1 \geq d_2$ .
  - **Observation:** For every extra letter the degrees it represents halves: 45, 22.5, 11.25, 6.125, ...
  - Return  $\min(d_2 - d_1, 360 + d_1 - d_2)$ .
- **Remark:** Only 29 characters are needed for the required precision.



Statistics: 144 submissions, 39 accepted, 21 unknown

# I: Imperfect Imperial Units

Problem Author: Abe Wits

- **Problem:** Convert values from one unit to another.
- **Solution:** For every query, do a breadth-/depth-first search, multiplying all conversion ratios on the path from the one to the other unit.
  - Runtime:  $\mathcal{O}(q \cdot n)$ , which is fast enough.
- **Faster solution:** Precalculate (or cache) the conversion ratio between all pairs of units.
  - Runtime:  $\mathcal{O}(n^2)$  to precalculate and  $\mathcal{O}(1)$  per query, so  $\mathcal{O}(n^2 + q)$ .
- Smallest/largest results are  $10^{\pm 303}$ , which fits in double (limit:  $9 \cdot 10^{\pm 307}$ ).
- **Pitfall:** In C++, people used `cout << fixed << setprecision(12)`. However, for values smaller than  $10^{-6}$ , this gives too low (*relative*) precision...

Statistics: 163 submissions, 31 accepted, 69 unknown

## D: Dividing DNA

Problem Author: Ragnar Groot Koerkamp

- **Problem:** Given a set of *forbidden* (*present*) intervals, partition  $[0, n)$  into as many disjoint (*absent*) intervals as possible.
- **Observation 1:** If an interval is forbidden, all shorter intervals are also forbidden!
- **Observation 2:** Each interval only needs to be *just* long enough to be allowed (*absent*).
- **Greedy solution:** Start with  $[0, 1)$ , and keep growing the interval until it is allowed, say  $[0, a)$ .
- Then start a new interval  $[a, a + 1)$ , and keep growing it until it is allowed, say  $[a, b)$ .
- Continue until reaching the end. This uses exactly  $n$  queries in total.



Statistics: 32 submissions, 10 accepted, 17 unknown

# C: Crashing Competition Computer

Problem Author: Jorke de Vlas

- **Problem:** What is the expected time to write  $c$  characters of code on a crash-prone computer?
- **Solution:** Dynamic programming over the length of the code.
- Time to code any  $x$  consecutive characters (without saving) =  
time to write  $x - 1$  characters + 1 + expected time needed to recover from crashing:

$$T(x) = T(x-1) + 1 + p \cdot (r + T(x)) = \frac{T(x-1) + 1 + p \cdot r}{1 - p} \quad \text{or} \quad T(x) = \frac{r+1}{p} \cdot ((1-p)^{-x} - 1)$$

- Calculate time to code all characters between position 0 and  $x$ ,  
minimising the total time by trying to click "Save" after character  $k$ :

$$DP(x) = \min \left( T(x), \min_{0 \leq k \leq x} \left( DP(k) + t + T(x-k) \right) \right)$$

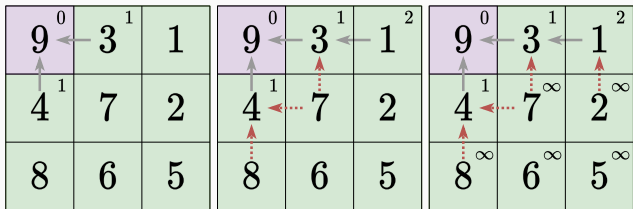
- Final answer is  $DP(c) + t$  (because we should save the code at the end, costing  $t$  time).
- Run-time complexity:  $\mathcal{O}(c^2)$

Statistics: 56 submissions, 10 accepted, 35 unknown

# K: Kiosk Construction

Problem Author: Reinier Schmiermann

- **Problem:** Find the optimal kiosk position for a given camping layout.
- **Solution:** Find the shortest path from every kiosk location  $k$  to every plot  $p$  ( $d(k, p)$ ), then calculate  $\min_k(\max_p(d(k, p)))$ .
  - But, doing  $n^2$  times BFS/DFS from every possible kiosk location to every plot is too slow ( $\mathcal{O}(n^3)$ ).
- **Optimisation:** Calculate distances the other way around: from every plot to every kiosk location.
  - The rules of walking between plots are fixed given a destination plot  $p$ , so do floodfill (BFS/DFS) starting from every destination plot  $p$ .
  - From a plot  $a$ , walk to neighbouring plots  $b$  if, according to the procedure, you can walk from  $b$  to  $a$  given the destination plot  $p$ .



- Run-time complexity:  $\mathcal{O}(n^2)$  (with  $n = h \cdot w$ ).

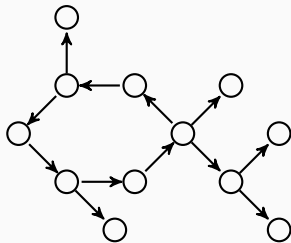
Statistics: 31 submissions, 8 accepted, 15 unknown



# H: House Numbering

Problem Author: Reinier Schmiermann

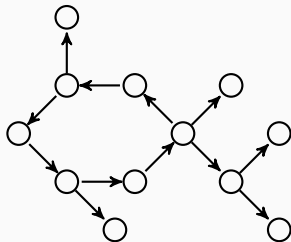
- **Problem:** Given a graph with  $n$  nodes and edges, and  $h$  house numbers for an edge, determine whether house numbers can be assigned such that there is no intersection where two edges start with the same house number.
- **Observation 1:** Every node can have at most one edge with house number 1 starting at it.
- **Observation 2:** Because the number of nodes is equal to the number of edges the graph contains exactly one cycle.
- **Observation 3:** The cycle has to be oriented clockwise or counterclockwise.
- **Observation 4:** The cycle has trees attached to it for which the house number 1 has to face outward.



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- **Problem:** Given a graph with  $n$  nodes and edges, and  $h$  house numbers for an edge, determine whether house numbers can be assigned such that there is no intersection where two edges start with the same house number.
- **Solution:**
  - First, find the cycle.
  - Orient cycle clockwise and propagate house numbers through trees. Check if this is valid, i.e., there are no two equal high house numbers at a node.
  - If not ok, orient cycle counter clockwise and check validity.
  - If one of the two works, print it. Otherwise, print "impossible".



Statistics: 31 submissions, 5 accepted, 15 unknown

# J: Jagged Skyline

Problem Author: Reinier Schmiermann

- **Problem:** Given  $w \leq 10\,000$  integers  $0 \leq h_i \leq 10^{18}$ , find the maximum in at most 12 000 queries: "Is integer  $h_i$  less than  $y$ ?"
- **First idea:** binary search per column:  $n \cdot \log_2(n)$  queries.
  - Only search the range above the current maximum?
  - In fact, we can only do 1.2 query per column. Most columns must be handled in 1 query exactly!
- **Better idea:** First test if a column is better than the current best. If not, skip it.
  - Problem: The maximum could increase in each column!
- **Solution:** Randomize the order of columns! Now, only  $\ln(w)$  increments are expected!
- Total expected queries:  $n + \ln(w) \cdot \log_2(n)$ .

Statistics: 140 submissions, 7 accepted, 80 unknown

# L: Lowest Latency

Problem Author: Reinier Schmiermann

- **Problem:** Given  $n = 10^5$  random points in  $V = [0, 10^9]^3$ , find the shortest distance between two points.
- Naive quadratic solution is too slow, and sweepline methods are complicated!
- To get a faster solution, make use of the fact that the input is random.
- **Observation:** Because the points are i.i.d. uniformly random, the answer is less than  $10^6$ .<sup>1</sup>
- **Solution:** Local bruteforce
  - Split the volume into  $100 \times 100 \times 100$  boxes of size  $10^7 \times 10^7 \times 10^7$ , and iterate over the pairs in each box.
  - Problem: The minimum distance may cross a boundary between boxes.
  - Solution: Iterate over all pairs of points in touching boxes as well.
  - Expected running time:  $\mathcal{O}(k \cdot (n/k)^2 + k) = \mathcal{O}(n^2/k + k)$ , where  $k$  is the number of boxes.
  - Note: due to the birthday paradox, there will practically always be a box with at least 2 points.

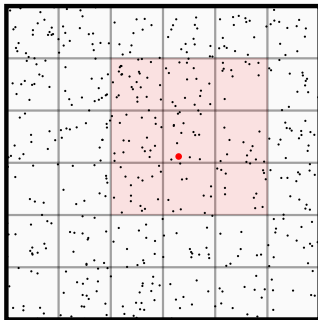
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<sup>1</sup>Or at least, *almost always* ;-)

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- **Solution:** Local bruteforce



# L: Lowest Latency

Problem Author: Reinier Schmiermann

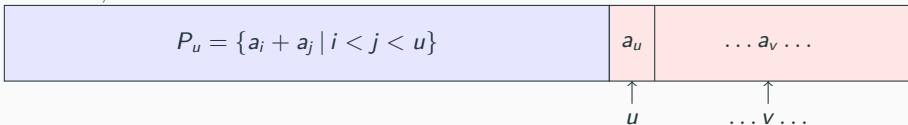
- **Problem:** Given  $n = 10^5$  random points in  $V = [0, 10^9]^3$ , find the shortest distance between two points.
- **Observation:** Because the points are i.i.d. uniformly random, the answer is less than  $10^6$ .
- **Alternative solution 1:** Divide & Conquer.
  - Sort the points by  $x$  and split into two groups of size  $n/2$ . Solve the two halves by recursively splitting in half-sized groups.
  - Let  $d$  be the best solution found recursively. Consider all pairs of points within distance  $d$  from the boundary to handle the *merge* step.
- **Alternative solution 2:** Sorted bruteforce.
  - Sort the points by  $x$ . The average  $x$ -distance between two points is  $10^9/n = 10^4$ .
  - Points  $> 100$  positions apart are expected to have distance  $> 100 \cdot 10^4 = 10^6$ .
  - Consider all pairs of indices  $(i, j)$  with  $|i - j| \leq 100$  for an  $\mathcal{O}(100n)$  solution.

Statistics: 63 submissions, 10 accepted, 35 unknown

# A: Adjusted Average

Problem Author: Ludo Pulles

- **Problem:** Given  $n \leq 1500$  integers  $a_i$ , remove at most  $k \leq 4$  of them to get an average as close as possible to the target  $\bar{x}$ .
- When removing exactly  $0 \leq \ell \leq k$  numbers, we need to find  $\ell$  integers with sum as close as possible to  $S_\ell = \sum_i a_i - \ell \cdot \bar{x}$ .
- For  $\ell \leq 2$ : iterate over all combinations  $\rightarrow \mathcal{O}(n^\ell/\ell!)$  time.
- For  $\ell = 3, 4$ : this is too slow so use *meet-in-the-middle*:



- For  $\ell = 3$ : loop over  $u = 1, \dots, n$ , update  $P_u$  and then take  $s \in P_u$  closest to  $S_\ell - a_u$ .
- Use an ordered set (BBST) for  $P_u$ , giving the time complexity  $\mathcal{O}(n^2 \log n)$ .
- For  $\ell = 4$ : For fixed  $u$ , loop over  $v$  with  $v > u$  and pick  $s \in P_u$  closest to  $S_\ell - a_u - a_v$ . This is still  $\mathcal{O}(n^2 \log n)$ .

Statistics: 25 submissions, 3 accepted, 13 unknown

# G: Grinding Gravel

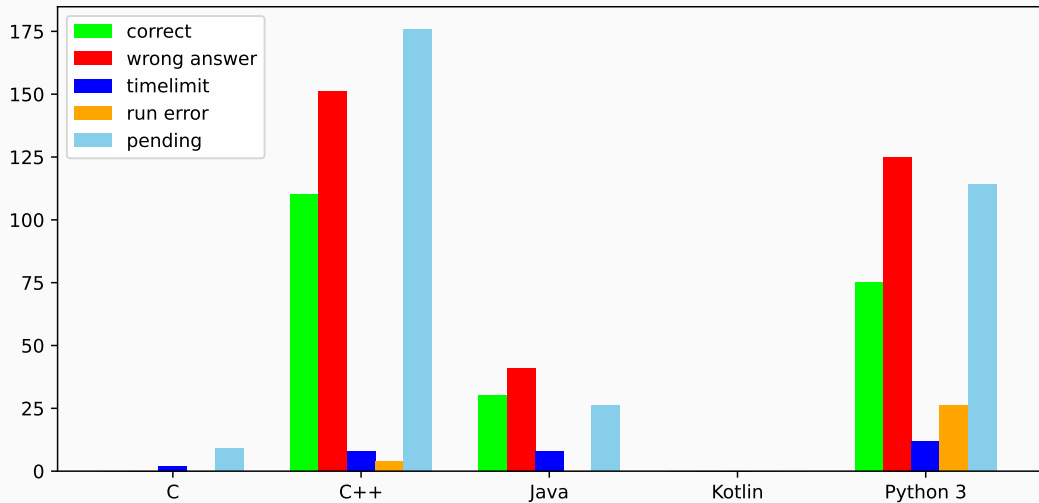
Problem Author: Daan van Gent, Onno Berrevoets

- **Problem:** Given  $n \leq 100$  integers, split them into groups of size  $k \leq 8$  making as few cuts as possible.
- **Equivalent problem:** Given  $n$  integers, partition them into as many groups as possible with sum a multiple of  $k$ .
- Greedy 1: Each number  $x \geq k$  is replaced by  $x \bmod k$ . Count the numbers with each remainder.
- Greedy 2: For  $x < k/2$ , we can pair up  $x$  and  $k - x$ . Each  $x = 0$  is its own group.
- We are left with at most 4 different values: 1 or 7, 2 or 6, 3 or 5, and at most one 4.
- Now, do a DP on state  $[c_1, \dots, c_{k-1}]$ , the counts for each remainder.
  - For each precomputed (minimal) subset with sum  $0 \bmod k$  remove it and recurse.
  - **Simpler alternative:** Merge the largest remainder with another one, and update the state.  $\rightarrow$  Too slow when counts are  $1 \times 4, 30 \times 5, 30 \times 6, 30 \times 7$ .
  - **Instead:** merge the least-occurring element with one of the others.
  - **Even simpler:** remove any one of the remaining elements. If this makes the total sum be  $0 \bmod k$ , add one.
- Instead of 4-deep nested loops, we can use a dictionary of tuples.

Statistics: 5 submissions, 1 accepted, 1 unknown



## Language stats



### Jury work

- 721 commits, of which 434 for the main contest
- 604 secret test cases (last year: 693) ( $= 50\frac{1}{3}$  per problem!)
- 165 jury solutions (last year: 177)
- The minimum<sup>2</sup> number of lines the jury needed to solve all problems is

$$14 + 3 + 5 + 1 + 4 + 4 + 27 + 34 + 14 + 15 + 18 + 4 = 143$$

On average 11.9 lines per problem, up from 9.6 in BAPC 2021 or 6.6 in preliminaries 2022

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<sup>2</sup>After codegolfing

## Thanks to:

### The proofreaders

Jaap Eldering  
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Mark van Helvoort  
Nicky Gerritsen  
Thomas Verwoerd

### The jury

Boas Kluiving  
Jorke de Vlas  
Ludo Pulles  
Maarten Sijm  
Ragnar Groot Koerkamp  
Reinier Schmiermann  
Ruben Brokkelkamp  
Wessel van Woerden

Want to join the jury? Submit to the Call for Problems of BAPC 2023 at:

<https://jury.bapc.eu/>