

# COCI 2017/2018

Round #5, January 20th

# Tasks

Task	Time limit	Memory limit	Score
Olivander	1 s	64 MB	50
Spirale	1 s	64 MB	80
Birokracija	1 s	64 MB	100
Karte	1 s	64 MB	120
Pictionary	1.5 s	64 MB	140
Planinarenje	1 s	256 MB	160
Total			650

Harry Potter has damaged his magic wand in a fight with Lord Voldemort. He has decided to get a new wand in Olivander's wand shop. On the floor of the shop, he saw *N* wands and *N* wand boxes. The lengths of the wands are, respectively,  $X_1, X_2...X_n$ , and the box sizes are  $Y_1, Y_2...Y_n$ . A wand of length *X* can be placed in a box of size *Y* if  $X \le Y$ . Harry wants to know if he can place all the wands in boxes so that each box contains exactly one wand. Help him solve this difficult problem.

#### INPUT

The first line of input contains the positive integer N ( $1 \le N \le 100$ ), the number from the task. The second line contains N positive integers  $X_i$  ( $1 \le X_i \le 10^9$ ), the numbers from the task. The third line contains N positive integers  $Y_i$  ( $1 \le Y_i \le 10^9$ ), the numbers from the task.

# OUTPUT

If Harry can place all the wands in boxes, output "DA" (Croatian for yes), otherwise output "NE" (Croatian for no).

## SCORING

In test cases worth 60% of total points, it will hold  $N \le 9$ .

# SAMPLE TESTS

input	input	input
3 7 9 5 6 13 10	4 5 3 3 5 10 2 10 10	4 5 2 3 2 3 8 3 3
output	output	output
DA	NE	DA

#### Clarification of the first test case:

Harry can place the wands in boxes. For example, he can place the wand of length 5 in a box of size 6, wand of length 7 in a box of size 13, and wand of length 9 in a box of size 10.

#### Clarification of the second test case:

Harry can't place the wands in boxes because the box of size 2 can't fit any of the wands.

Little Stjepan often likes to go out with his friends and have fun in a popular nightclub in Zagreb. However, Stjepan sometimes drinks too much soda and gets light headed from all the sugar. Last night was an example of this, which is why Stjepan had the same image in his mind the whole time. It was a scribble of number spirals of some sort. Since he can't quite remember what the image looked like, but can describe it, he is asking you to reconstruct it for him.

Stjepan recalls that the image was of the shape of a table consisting of numbers written in N rows and M columns. Also, he recalls that there were K spirals in that table. For each spiral, the starting position is known, as well as the direction it's moving in, which can be clockwise and counter-clockwise. An example is shown in the images below. The spirals created Stjepan's image in exactly 10<sup>100</sup> steps in the following way:

- 1. Initially, the table is empty, and each spiral is in its own starting position.
- 2. In each following step, each spiral moves to its next position. It is possible that, at times, the spirals leave the boundaries of the table, but also to return within it.
- 3. After exactly 10<sup>100</sup> steps, for each field in the table, the final value is the value of the earliest step in which one of the spirals touched that field.

3	2	9
4	1	8
5	6	7

9	2	3
8	1	4
7	6	5

Image 1: a spiral moving counter-clockwise

Image 2: a spiral moving clockwise

#### INPUT

The first line of input contains positive integers N, M ( $1 \le N$ ,  $M \le 50$ ) and K ( $1 \le K \le N^*M$ ). Each of the following K lines contains three positive integers  $X_i$ ,  $Y_i$  and  $T_i$  ( $1 \le X \le N$ ,  $1 \le Y \le M$ ,  $0 \le T \le 1$ ), the starting position of the *i*<sup>th</sup> spiral and its direction (0 - clockwise, 1 - counter-clockwise). No two spirals will begin in the same field.

#### OUTPUT

You must output *N* lines with *M* numbers, representing the table after each spiral makes  $10^{100}$  steps.

#### SCORING

In test cases worth 50% of total points, it will hold: N=M i K=1 and  $X_i=Y_i=\lfloor \frac{N+1}{2} \rfloor$ , i.e.  $X_i$  and  $Y_i$  will be equal to the integer division of N+1 with 2.

#### SAMPLE TESTS

input	input	input
3 3 1 2 2 0	3 3 1 2 2 1	3 3 2 1 1 0 1 2 0
output	output	output

#### Clarification of the third test case:

A10	A11, B10	A12, B11	A13, B12	B13
A9	A2, B9	A3, B2	A14, B3	B14
A8	A1, B8	A4, B1	A15, B4	B15
A7	A6, B7	A5, 6	A16, B5	B16
A20, B21	A19, B20	A18, B19	A17, B18	B17

For simplicity's sake, the letter A was added to the numbers from the first spiral, and the letter B to the numbers from the second spiral. Only the first 20 steps of the first spiral are shown, and 21 steps of the second spiral. The fields in gray are the fields from the table we're interested in, all other fields are out of the table's bounds, but are shown to illustrate the way the spirals move outside of the table.

Mirko has become CEO of a huge corporation. This corporation consists of N people, labeled from 1 to N, and Mirko is labeled 1. The corporation is structured so that each employee (except Mirko) has a boss, and we say that employee is an assistant to that boss. Each boss can have multiple assistants, but still reports to their boss. This holds for everyone except Mirko, who is at the top of the pyramid and doesn't have a boss, but has his assistants.

When Mirko gets a task from the investors, he then delegates that task to his assistant with the minimal number. This assistant then delegates the task to their assistant with the minimal number, and this process repeats until the task is given to an unlucky person without an assistant, who then must do the task.

This is when the real problems begin. The person that did the task gets paid 1 coin, the boss of that person gets 2 coins, the boss of that person gets 3 coins, and so on, all the way to Mirko, who gets as many coins as there are people in this sequence. After that, the employee that really did the job realizes how unfair the system is and quits their job out of principle.

When it comes to doing the next task in the corporation, there is a person less, so maybe the paychecks are less, but the work must continue. Tasks are piling up, so the whole procedure (assigning a new task, doing it, dividing paychecks and the person doing the task quitting) repeats until Mirko is left alone in the corporation and does his first (also his last) task.

Of course, Mirko will have amassed quite a fortune until then, but he also wants to know how much money each of the employees earned.

#### INPUT

The first line of input contains the positive integer N ( $2 \le N \le 2.10^5$ ), the number of employees (including Mirko).

The following line contains N - 1 positive integers  $a_2$ ,  $a_3$ ,  $a_4$ , ...,  $a_n$  ( $1 \le a_i \le i$ ), where  $a_i$  denotes the boss of employee *i*.

#### OUTPUT

You must output a single line consisting of *N* numbers, the  $i^{th}$  number corresponding to the amount of money earned by the  $i^{th}$  employee.

#### SCORING

In test cases worth 50% of total points, it will hold  $2 \le N \le 5000$ .

input	input
3 1 1	5 1 2 2 4
output	output
5 1 1	13 8 1 3 1

#### Clarification of the second test case:

Mirko assigns the first task to employee 2, who then assigns it to employee 3, who then does it. For this, employee 3 gets 1 coin, employee 2 gets 2 coins, and employee 1 (Mirko) gets 3 coins. Employee 3 then quits.

Mirko assigns the second task to employee 2, who then assigns it to employee 4 (because employee 3 quit), who then assigns it to employee 5, who then does it. For this, employee 5 gets 1 coin, employee 4 gets 2 coins, employee 2 gets 3 coins, and employee 1 gets 4 coins. Employee 5 then quits.

The procedure is repeated for a total of 5 tasks.

In total, Mirko gets 13 coins, employee 2 gets 8, employee 4 gets 3, and employee 3 and 5 each get 1 coin.

... Take these cards, I tell ya, my cousin from Sweden sent them when the hunger for war was at its peak, and we stayed playing rummy in the trenches.

Daniel: "Are we playin' rummy, eh?"

Domagoj: "Well ok then."

Daniel: "Watch this now. You have a deck of *N* cards, where the  $i^{th}$  card has a claim written on it in the form of 'At least  $a_i$  cards beneath this one contain a false claim.' You have to shuffle them so that exactly *K* cards contain a false claim."

Domagoj: "You destroy me in this game every time, where did you get these cards, son?!"

Daniel: "Ah, my old man organizes card tournaments, so each day he gives me free cards, ten bucks per deck."

Your task is to help Domagoj solve Daniel's task. Output the order in which Domagoj must place the cards. It is also possible that this is a bad joke on Daniel's part, and that there is no possible order to place the cards in. In that case, output -1.

#### INPUT

The first line contains integers *N* and *K* ( $1 \le N \le 5 \cdot 10^5$ ,  $0 \le K \le N$ ). The *i*<sup>th</sup> of the following *N* lines contains an integer  $a_i$  ( $0 \le a_i \le 5 \cdot 10^5$ ).

# OUTPUT

If the required order does not exist, output -1.

Otherwise, in a single line, output *N* integers separated by spaces, the numbers on the cards in the order **from the top to the bottom** of the deck. If there are multiple solutions, output any.

#### SCORING

In test cases worth 30% of total points, it will hold  $N \le 16$ . In additional test cases worth 40% of total points, it will hold  $N \le 2000$ .

input	input	input
4 2	5 3	6 4
1	2	0
2	1	2
2	3	5
3	0	2
	3	0
		1
output	output	output
2 3 1 2	3 3 0 1 2	-1

#### Clarification of the second test case:

For simplicity's sake, we'll label the cards as true/false, depending on the claims written on them. Beneath the fifth card there are 0 false cards, so it is false.

Beneath the fourth card there is 1 false card, so it is true.

Beneath the third card there is 1 false card, so it is true.

Beneath the second card there is 1 false card, so it is false.

Beneath the first card there are 2 false cards, so it is false.

Therefore, we have a total of 3 false cards.

There is a planet, in a yet undiscovered part of the universe, with a country inhabited solely by mathematicians. In this country, there are a total of N mathematicians, and the interesting fact is that each mathematician lives in their own city. Is it also interesting that no two cities are connected with a road, because mathematicians can communicate online or by reviewing academic papers. Naturally, the cities are labeled with numbers from 1 to N.

Life was perfect until one mathematician decided to write an academic paper on their smartphone. The smartphone auto-corrected the word "self-evident" to "Pictionary" and the paper was published as such. Soon after, the entire country discovered pictionary and wanted to meet up and play, so construction work on roads between cities began shortly.

The road construction will last a total of *M* days, according to the following schedule: on the first day, construction is done on roads between all pairs of cities that have *M* as their greatest common divisor. On the second day, construction is done on roads between all pairs of cities that have *M*-1 as their greatest common divisor, and so on until the  $M^{th}$  day when construction is done on roads between all pairs of cities that are co-prime. More formally, on the *i*<sup>th</sup> day, construction is done on roads between cities *a* and *b* if gcd(a, b) = M - i + 1.

Since the mathematicians are busy with construction work, they've asked you to help them determine the minimal number of days before a given pair of mathematicians can play pictionary together.

#### INPUT

The first line of input contains three positive integers *N*, *M* and *Q* ( $1 \le N$ ,  $Q \le 100\ 000$ ,  $1 \le M \le N$ ), the number of cities, the number of days it takes to build the roads, and the number of queries.

Each of the following Q lines contains two distinct positive integers A and B ( $1 \le A, B \le N$ ) that denote the cities of the mathematicians who want to find out the minimal number of days before they can play pictionary together.

# OUTPUT

The  $i^{th}$  line must contain the minimal number of days before the mathematicians from the  $i^{th}$  query can play pictionary together.

# SCORING

In test cases worth 40% of total points, it will hold  $N \le 1000$ ,  $Q \le 100\ 000$ .

input	input	input
8 3 3 2 5 3 6 4 8	25 6 1 20 9	9999 2222 2 1025 2405 3154 8949
output	output	output
3 1 2	4	1980 2160

#### Clarification of the first test case:

On the first day, road (3, 6) is built. Therefore the answer to the second query is 1. On the second day, roads (2, 4), (2, 6), (2, 8), (4, 6) and (6, 8) are built. Cities 4 and 8 are now connected (it is possible to get from the first to the second using city 6).

On the third day, roads between relatively prime cities are built, so cities 2 and 5 are connected.

#### Clarification of the second test case:

On the second day, road (20, 15) is built, whereas on the fourth day, road (15, 9) is built. After the fourth day, cities 20 and 9 are connected via city 15.

Mirko and Slavko like to hike together. Mirko likes mountain peaks, and Slavko likes valleys. Because of this, every time they climb to a peak, Slavko decides which valley they are going to descend to, given that a trail exists to it. Similarly, in each valley, Mirko decides which peak they are going to climb up to. It will never be possible to directly climb from one peak to another, or to get from one valley to another valley. In order to make the hiking as fun as possible, they never visit the same spot twice (whether it's a peak or a valley). Once they reach a spot that only leads to spots they've visited before, they call the mountain rangers to pick them up. If the final spot is a peak, Mirko wins, and if it is a valley, Slavko wins.

Naturally, you must already know what your task is: If we assume that both of them play optimally, who wins? Answer this question for all initial peaks.

#### INPUT

The first line contains two positive integers, *N* and *M* ( $1 \le N \le 5000$ ,  $1 \le M \le \min(5000, N \cdot N)$ ). Here *N* denotes the number of peaks and valleys (therefore, there are *N* peaks and *N* valleys), and *M* denotes the number of hiking trails.

Each of the following *M* lines contains two positive integers:  $v_i$  and  $d_i$  ( $1 \le v_i$ ,  $d_i \le N$ ) that denote there is a trail between peak  $v_i$  and valley  $d_i$ .

Between each peak and valley, there will exist at most one trail.

## OUTPUT

You must output *N* lines. The *i*<sup>th</sup> line denotes the winner if the starting point is peak *i*.

# SCORING

In test cases worth 30% of total points, it will hold  $1 \le N \le 10$  and  $1 \le M \le N \cdot N$ . In test cases worth an additional 20% of total points, for each two connected spots, there will exist a unique path between them. In other words, the graph will be a forest.

input	input	input
2 3 1 2 2 2 2 1	4 5 2 2 1 2 1 1 1 3 4 2	4 5 1 2 1 3 2 2 2 3 4 1
output	output	output
Slavko Slavko	Slavko Mirko Mirko Mirko	Slavko Slavko Mirko Slavko

#### Clarification of the second test case:

Starting from the first peak, Slavko can choose to go to the first valley, so Slavko wins.

Starting from the second peak, Slavko can choose to go to the second valley, after which Mirko wins by choosing to go to the fourth peak.

Starting from the third peak, there are no trails, so Mirko wins.

Starting from the fourth peak, Slavko must choose to go to the second valley, after which Mirko wins by choosing the second peak.