NCPC 2021
Presentation of solutions

2021-10-09

Problems prepared by

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- Antti Laaksonen (CSES)
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- Bergur Snorrason (University of Iceland)

K — Knot Knowledge

Problem

You are given a list of numbers and then another list of the same numbers, except one number has been removed. What number was removed?

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3. Let the first list of numbers be $x_1, x_2, \ldots x_n$ and the second list be $y_1, y_2 \ldots y_{n-1}$.

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3. Let the first list of numbers be $x_1, x_2, \ldots x_n$ and the second list be $y_1, y_2 \ldots y_{n-1}$.
4. The answer is then

$$\sum_{k=1}^{n} x_k - \sum_{k=1}^{n-1} y_k.$$

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Statistics at 4-hour mark: 248 submissions, 196 accepted, first after 00:01

J — Joint Jog Jam

Problem

Compute the maximum distance between two people moving in straight line segments at constant speed.

Solution

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1. The answer is simply the max of the distance between the starting points and the distance between the ending points. Let us see why...

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2. Useful trick: by viewing everything from the reference frame of the first runner, we can instead assume that the first person is stationary at $(0, 0)$.
3. Second person is at some position $(x(t), y(t)) = (x_0 + t \cdot x_\delta, y_0 + t \cdot y_\delta)$ at time t.

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4. Squared distance at time t is then $x(t)^2 + y(t)^2 = (x_0 + t \cdot x_\delta)^2 + (y_0 + t \cdot y_\delta)^2$

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4. Squared distance at time t is then $x(t)^2 + y(t)^2 = (x_0 + t \cdot x_\delta)^2 + (y_0 + t \cdot y_\delta)^2$
5. This is a convex quadratic function in t, so it is maximized at $t = t_{\text{min}}$ or $t = t_{\text{max}}$.

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Statistics at 4-hour mark: 274 submissions, 189 accepted, first after 00:10

L — Locust Locus

Problem

A pair of cicada species appear every a and b years, and was last observed in year y. Find the next year they will appear together again.

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1. For each year $z = y + 1, y + 2, y + 3, \ldots$:
   - Check if $z - y$ is divisible by both a and b.
   - Break when first such z is found.

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3. So this takes $O(a \cdot b)$ time which is fast enough because a and b are very small.

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Solution 2

1. The joint period of the two species is the least common multiple of a and b.

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Statistics at 4-hour mark: 374 submissions, 185 accepted, first after 00:02

G — Grazed Grains

Problem

Given $n \leq 10$ circles of radius ≤ 10 and with centers in $[0, 10] \times [0, 10]$, approximate the area of their union, up to a factor 1 ± 0.1.

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   Not too hard, but a bit of code, and there is a simpler solution: use sampling.

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2. Sample r uniformly random points in $[-10, 20] \times [-10, 20]$.
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3. If x of the r points are inside some circle, we estimate the area as $\frac{x}{r} \cdot 30^2$.

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4. Analysis (not needed to solve the problem): can prove that you expect a relative error around $24/\sqrt{r}$. If $r > 100$k this starts becoming small enough, and with $r = 1$ million the sampling error is very unlikely to be too large.

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Statistics at 4-hour mark: 267 submissions, 117 accepted, first after 00:07

A — Antenna Analysis

Problem

Given integers $x_1, \ldots, x_n$, find, for each i, the maximum of $|x_i - x_j| - c|i - j|$ over $j \leq i$.

Solution

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Given integers $x_1, \ldots, x_n$, find, for each i, the maximum of $|x_i - x_j| - c|i - j|$ over $j \leq i$.

Solution

1. Key observation: dropping the absolute values does not increase the answer.

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Solution

1. Key observation: dropping the absolute values does not increase the answer.
2. Simplify problem: drop the absolute values and maximize $(x_i - x_j) - c(i - j) = (x_i - c \cdot i) + (-x_j + c \cdot j)$.

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   - Keep track of largest value y of $-x_j + c \cdot j$ seen so far.

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   - Answer to simplified problem is $x_i - c \cdot i + y$.

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3. To solve original problem, separately solve variant for $(x_j - x_i) - c(i - j)$ the same way and take largest of the two.

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4. Time complexity $O(n)$.

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5. More complicated solutions using balanced search trees or range trees also possible.

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Statistics at 4-hour mark: 626 submissions, 88 accepted, first after 00:04

D — Deceptive Directions

Problem

Get $w \times h$ grid map and a shortest sequence of NWSE steps to reach some treasure. But all the steps have been replaced by wrong ones. Where could the treasure be?

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1. Compute distance $d[p]$ from start to every position p in the maze using BFS.

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2. Forbid all steps that could not have been used by the correct instruction sequence:

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3. Run BFS again with these steps forbidden.

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   - All reached positions at same distance as instruction length are possible treasure locations.

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4. Time complexity: $O(w \cdot h)$.

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3. Run BFS again with these steps forbidden.
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4. Time complexity: $O(w \cdot h)$.

Statistics at 4-hour mark: 325 submissions, 55 accepted, first after 00:38

F — Fortune From Folly

Problem

In infinite random binary sequence $x_1, x_2, x_3, \ldots$ where each $x_i = 1$ with probability p (independently), what is expected first value of i such that $x_i + x_{i-1} + \ldots + x_{i-n+1} \geq k$?

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Solution

1. At any point, only the n most recent x_i's matter.

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Solution

1. At any point, only the n most recent x_i's matter.
2. Let $E_{z_1 z_2 z_3 \ldots z_n}$ be expected #steps until k ones, if most recent x_i's are $z_1, \ldots, z_n$.

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3. This is a system of linear equations in the 2^n unknowns $E_{z_1 z_2 z_3 \ldots z_n}$.
   - Solve using Gaussian elimination to find our answer $E_{00 \ldots 0}$.

F — Fortune From Folly

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In infinite random binary sequence $x_1, x_2, x_3, \ldots$ where each $x_i = 1$ with probability p (independently), what is expected first value of i such that $x_i + x_{i-1} + \ldots + x_{i-n+1} \geq k$?

Solution

1. At any point, only the n most recent x_i's matter.
2. Let $E_{z_1 z_2 z_3 \ldots z_n}$ be expected #steps until k ones, if most recent x_i's are $z_1, \ldots, z_n$.
   - If $\sum_{i=1}^{n} z_i \geq k$ then $E_{z_1 z_2 z_3 \ldots z_n} = 0$.
   - Otherwise $E_{z_1 z_2 z_3 \ldots z_n} = 1 + p \cdot E_{z_2 z_3 \ldots z_n 1} + (1-p) \cdot E_{z_2 z_3 \ldots z_n 0}$.
3. This is a system of linear equations in the 2^n unknowns $E_{z_1 z_2 z_3 \ldots z_n}$.
   - Solve using Gaussian elimination to find our answer $E_{00\ldots0}$.
   - Time complexity $O(2^{3n})$.

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   - $z_2 z_3 \ldots z_n 0 \quad \longleftrightarrow \quad (\texttt{Z} >> 1)\ \texttt{OR}\ (\texttt{b} << (\texttt{n} - 1))$

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   - $z_2 z_3 \ldots z_n 0 \quad \longleftrightarrow \quad \texttt{(Z >> 1) OR (b << (n - 1))}$

Statistics at 4-hour mark: 57 submissions, 27 accepted, first after 00:31

C — Customs Controls

Problem

Given a vertex-weighted graph, color k vertices red and $n - k$ vertices blue such that every shortest path from 1 to n has a monochromatic edge.

Solution

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1. Dijkstra's algorithm finds all edges that are part of some shortest path from 1 to n.

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1. Dijkstra's algorithm finds all edges that are part of some shortest path from 1 to n.
2. These edges form a directed acyclic graph. Find a topological ordering.

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2. These edges form a directed acyclic graph. Find a topological ordering.
3. Color the first k vertices in the ordering red, and the remaining ones blue:
   - A shortest path from 1 to n now only switches between red and blue once, so every shortest path on 3 or more vertices must have a monochromatic edge.

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   - Since graph is vertex-weighted, edge from 1 to n is the *only* shortest path.
   - We only need to make sure 1 and n get the same color.

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   - Always possible, except if $n = 2$ and $k = 1$.

Statistics at 4-hour mark: 67 submissions, 15 accepted, first after 01:39

H — Hiring Help

Problem

Given productivity values ℓ_i, f_i of n coders and productivity ℓ, f of consultant for t-hour long project, is there a weighted average of coders such that $\ell_{\text{avg}} \geq \ell/t$ and $f_{\text{avg}} \geq f/t$? Handle many queries like this, interleaved with some of the n coders leaving.

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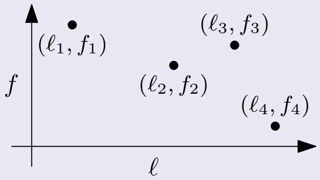
Geometric View

1. The coders are a set of points P in 2D.



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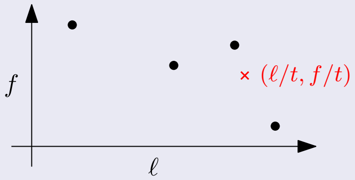
Geometric View

1. The coders are a set of points P in 2D.
2. The consultant query is another point q in 2D.



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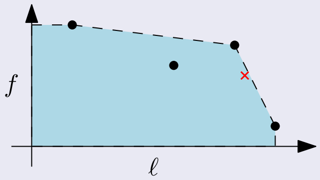
1. The coders are a set of points P in 2D.
2. The consultant query is another point q in 2D.
3. The weighted average exists if and only if q is below the upper side of the convex hull of P.



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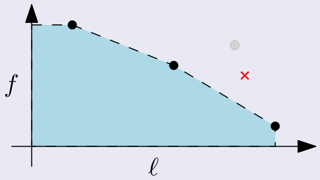
Geometric View

1. The coders are a set of points P in 2D.
2. The consultant query is another point q in 2D.
3. The weighted average exists if and only if q is below the upper side of the convex hull of P.
4. Coders leaving corresponds to points being removed, leading to the convex hull changing.



H — Hiring Help

Reformulated Problem

Given set of points (x, y), maintain upper side of its convex hull, under removals of points and queries about whether other points (x^*, y^*) are below the hull.

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Given set of points (x, y), maintain upper side of its convex hull, under removals of points and queries about whether other points (x^*, y^*) are below the hull.

Solution

1. The hull is a piecewise linear function, represent it as a sorted set of points $(x_1, y_1), (x_2, y_2), \ldots, (x_t, y_t)$ where $x_1 < x_2 < \ldots x_t$ and $y_1 > y_2 > \ldots y_t$.

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2. For a query (x^*, y^*), find index i such that $x_{i-1} < x^* \leq x_i$ and check if (x^*, y^*) is below line from (x_{i-1}, y_{i-1}) to (x_i, y_i).

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3. Handling removals can be done,

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Given set of points (x, y), maintain upper side of its convex hull, under *additions* of points and queries about whether other points (x^*, y^*) are below the hull.

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3. Handling removals can be done, but if we instead run the events in reverse order, the removals become *additions*, which are easier to handle.

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Given set of points (x, y), maintain upper side of its convex hull, under *additions* of points and queries about whether other points (x^*, y^*) are below the hull.

Handling additions

1. If point to add is outside current hull, add it to our current set of points.

H — Hiring Help

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2. Remove any concavities formed to left and right of new point.

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4. Not an issue: once a point is discarded, it can never be added back, so total number of removals for all events is $\leq n$.
5. $O((n + e) \log n)$ total time complexity.

Statistics at 4-hour mark: 24 submissions, 7 accepted, first after 01:42

I — Intact Intervals

Problem

Given circular array a, how many ways can it be separated into two or more intervals such that array b can be obtained by permuting each interval separately?

Problem

Given circular array a, how many ways can it be separated into two or more intervals such that array b can be obtained by permuting each interval separately?

Solution for the **non-circular** case

1. Let $A_i = a_1, \ldots, a_i$ be the prefix of first i values of a.

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2. Let s be the number of indices $1 \leq i < n$ such that A_i is a permutation of B_i.

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3. Then the number of ways is $2^s - 1$:

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   - breaking up a at any non-empty subset of these indices results in a valid separation

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4. To find s quickly, can use a permutation-invariant hash function.

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   - Assign a random hash value $h(x)$ to each array value x.
   - Define hash $h(A_i)$ of a prefix to be $\sum_{j=1}^{i} h(a_j)$.
   - If no hash collisions then A_i is a permutation of B_i if and only if $h(A_i) = h(B_i)$.

Problem

Given circular array a, how many ways can it be separated into two or more intervals such that array b can be obtained by permuting each interval separately?

Solution for the circular case

1. For each hash value z, let $s(z)$ be number of indices $0 \leq i < n$ such that $h(A_i) - h(B_i) = z$.

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   - For each z, taking any subset of at least 2 of the $s(z)$ indices is a valid separation.
   - Taking two indices with different values of $h(A_i) - h(B_i)$ gives an invalid separation.
3. Results in an $O(n)$ time solution, assuming $O(1)$-time dictionaries which can be used to store the frequency of each hash value.

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   - Taking two indices with different values of $h(A_i) - h(B_i)$ gives an invalid separation.
3. Results in an $O(n)$ time solution, assuming $O(1)$-time dictionaries which can be used to store the frequency of each hash value.

Statistics at 4-hour mark: 11 submissions, 5 accepted, first after 01:51

B — Breaking Bars

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   Split one 3x5 as `3x2+3x3` and the other as `1x5+2x5` to get away with two splits.

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6. But this *does* give upper bound on number of breaks that may be needed (it is 9).

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Statistics at 4-hour mark: 17 submissions, 1 accepted, first after 01:13

E — Eavesdropper Evasion

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3. Generalize it to at most 2 intercepted messages

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4. By placing the two shortest messages first and last, we get the optimal solution.

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1. Key observation: we can view this as having 2 separate channels, and we want at most 1 intercepted message in each channel (not immediately obvious!).

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Statistics at 4-hour mark: 19 submissions, 0 accepted

M — Marvelous Marathon

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Find a path of length x subject to certain constraints in a $2 \times m$ grid so that the sum of the values in the cells of the path is maximized.

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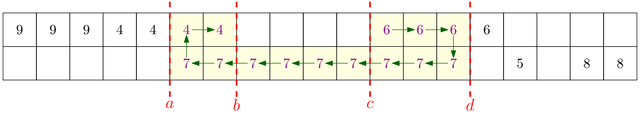
Formalized version of problem

Find integers $0 \leq a \leq b \leq c \leq d \leq m$ such that $2(b-a)+(c-b)+2(d-c)=x$.



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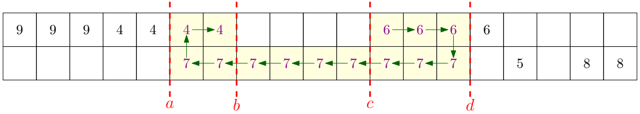
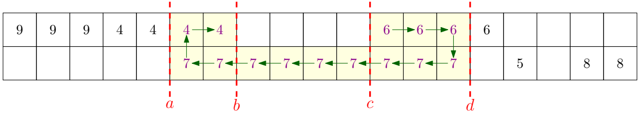
$$\text{Maximize} \left\{ \begin{array}{c} \text{sum of cells of one row in range } [a, d) \\ \text{plus} \\ \text{sum of cells of other row in ranges } [a, b) \text{ and } [c, d) \end{array} \right\}$$



M — Marvelous Marathon

Solution outline

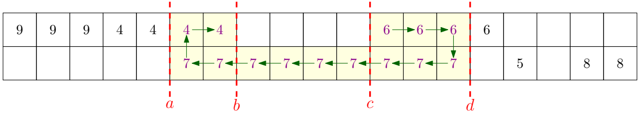
1. m is very large, cannot loop over all cells



M — Marvelous Marathon

Solution outline

1. m is very large, cannot loop over all cells
2. Large parts of grid look the same because only n segments



M — Marvelous Marathon

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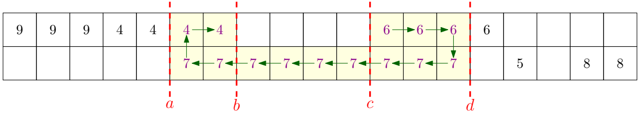

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2. Large parts of grid look the same because only n segments
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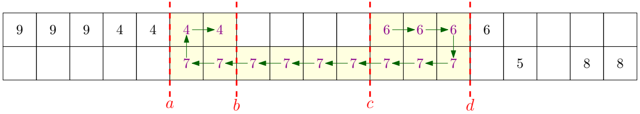
4. We focus here only on the hardest case with 2 U-turns.



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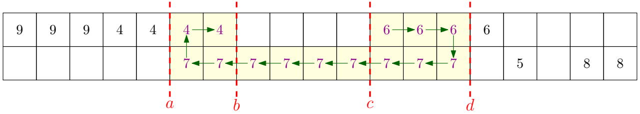
Insight 1

1. We can assume solution has the gap in the lower half
   - Run solution again on flipped input to cover opposite case



M — Marvelous Marathon

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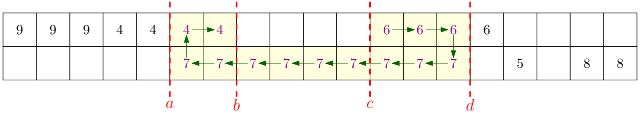
2. There is an optimal solution where a or d is at a segment endpoint (or 0 or m). Otherwise we could decrease (or increase) both a and d with 1 until either



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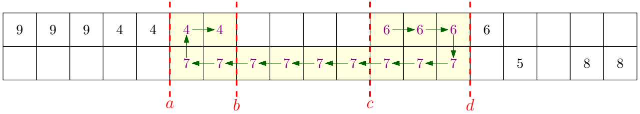

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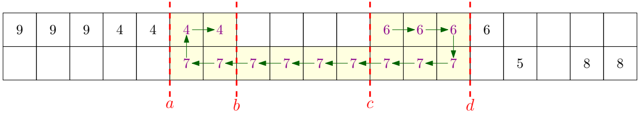

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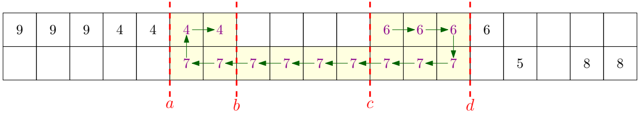
3. We can assume a is the endpoint
   - Run solution again on reversed input to cover opposite case.



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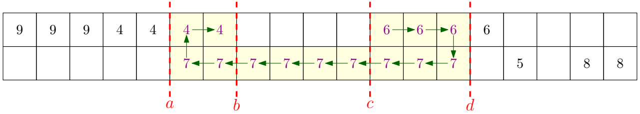

M — Marvelous Marathon

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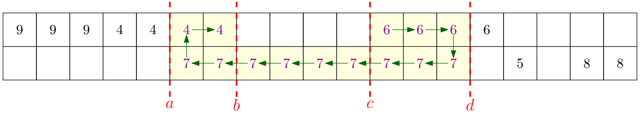

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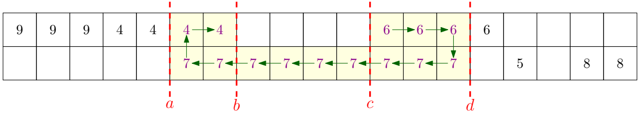
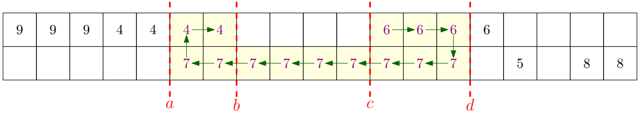
   - a and c are segment endpoints
3. Will focus on the first of these; the other must also be solved, but is done in a very similar fashion.



M — Marvelous Marathon

Sliding

1. Fix some a and b. ($O(n^2)$ possible choices.)



M — Marvelous Marathon

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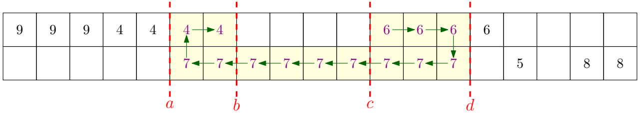
2. Set $c = b$ (or $c = b + 1$ if x is odd) and $d = a + \lceil x/2 \rceil$



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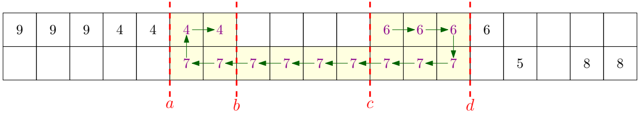

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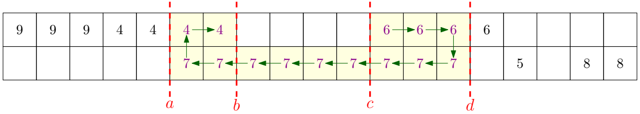
   - Since grid values rarely change value, we can slide many steps at a time.



M — Marvelous Marathon

Sliding

1. Fix some a and b. ($O(n^2)$ possible choices.)
2. Set $c = b$ (or $c = b + 1$ if x is odd) and $d = a + \lceil x/2 \rceil$
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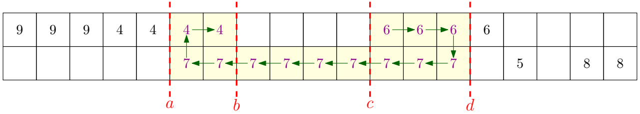

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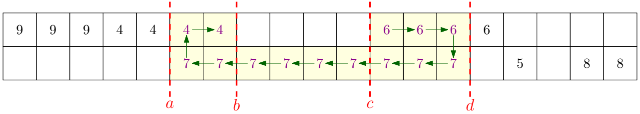

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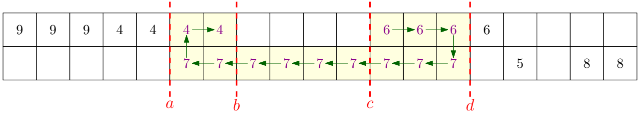

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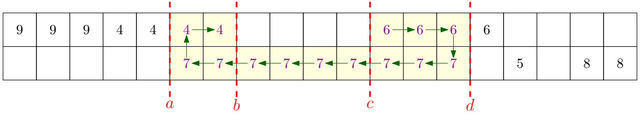

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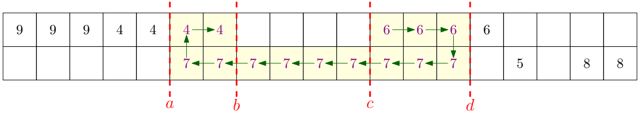
Statistics at 4-hour mark: 0 submissions, 0 accepted



Results!