NCPC 2020
Presentation of solutions

2020-11-07

Problems prepared by

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- Simon Lindholm (Vårdinnovation)
- Ulf Lundström (Excillum)
- Jimmy Mårdell (Spotify)
- Johan Sannemo (Huawei)
- Bergur Snorrason (University of Iceland)
- Pehr Söderman (Kattis)

M — Methodic Multiplication

Problem

Multiply two natural numbers in Peano arithmetic.

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Axiomatic Solution

```
main :- read_val(X), read_val(Y), multiply(X, Y, Z), print(Z).

multiply(_, 0, 0).
multiply(X, s(Y), Z) :- multiply(X, Y, W), add(W, X, Z).

add(X, 0, X).
add(X, s(Y), Z) :- add(X, Y, W), Z = s(W).

read_val(0)    :- peek_code(C), code_type(C, space), !, get_char(_).
read_val(s(X)) :- get_char(C), C == 'S', !, read_val(X).
read_val(X)    :- read_val(X).
```

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Multiply two natural numbers in Peano arithmetic.

Non-Axiomatic Solution

```
x = input().count('S')
y = input().count('S')
z = x*y
print('S('*z + '0' + ')'*z)
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Statistics at 4-hour mark: 317 submissions, 188 accepted, first after 00:01

C — Coin Stacks

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3. Easy to prove that if (A) and (B) hold before such a move, they also hold after.

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Statistics at 4-hour mark: 555 submissions, 146 accepted, first after 00:07

A — Array of Discord

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   E.g. cannot change 124 into 024 but can change 4 into 0.

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Statistics at 4-hour mark: 811 submissions, 125 accepted, first after 00:04

D — Dams in Distress

Problem

We get a rooted tree forming a system of $n \leq 200\,000$ dams. Overflowing a dam causes it to break and release all its water downstream. What is minimum amount of water we need to add at one dam in order for w units of water to reach the root?

Solution

1. For each dam i compute how much water $f(i)$ is needed if we add water at i.

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   - Need to add $c_i - u_i$ water to break the dam, this causes c_i water to go upstream.
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Gives equation $f(i) = c_i - u_i + \max(0, f(p_i) - c_i)$.

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4. Complexity $O(n)$ – compute top-down so $f(p_i)$ is known when computing $f(i)$.

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Statistics at 4-hour mark: 270 submissions, 90 accepted, first after 00:32

G — Gig Combinatorics

Problem

Given sequence of $n \leq 10^6$ digits 1/2/3, count how many subsequences have the form "12+3" ("2+" means 1 or more twos)

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3. Answer is sum of $p(i)$ over all positions i where there we have a 3.
4. Time complexity $O(n)$.

Statistics at 4-hour mark: 437 submissions, 78 accepted, first after 00:04

J — Joining Flows

Problem

Given $k \leq 10$ faucets with different temperatures and adjustable flow levels, determine if they can be combined to produce given flow level and temperature.

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Statistics at 4-hour mark: 128 submissions, 46 accepted, first after 00:44

F — Film Critics

Problem

We have one movie and n critics with opinions $x_1, \ldots, x_n$ on how good it is.

If current review average of the movie exceeds a reviewers opinion they will score it 0, otherwise they will score it m.

Order the critics so that the film ends up getting review average exactly k/n.

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1. Each review is either *positive* (score m) or *negative* (score 0).

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Order the critics so that the film ends up getting review average exactly k/n.

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2. If there are p positive reviews, final review average is pm/n.
3. So k must be divisible by m (otherwise `impossible`), and we need exactly $p = k/m$ positive reviews.

F — Film Critics

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2. Build the answer iteratively. Each iteration, two candidates for next reviewer:
   - lowest remaining x_i from among the p largest, or
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Statistics at 4-hour mark: 62 submissions, 21 accepted, first after 00:43

K — Keep Calm and Carry Off

Problem

Given two 1 000 000-digit integers A and B, find the smallest non-negative integer X such that $A + X$ and $B - X$ (or $A - X$ and $B + X$) can be added without carry.

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Solution

Simplifying assumptions:

1. A and B have the same number of digits (or zero-pad)
2. it is always the first integer we add to (try both options; take the best one)

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2. We must increment a_i and decrement b_i (mod 10) until there is no longer carry.

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4. If i is the leftmost digit causing carry, turning it to 0 will turn all remaining digits to 0 as well and get rid of any carries there.
5. Lets us compute $A + X$ easily, subtract A to get X.

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1. Caveat: turning the leftmost carry a_i into a 0 causes a_{i-1} to increase by 1, can cause a new carry in the previous digit.

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2. Any preceding sequence of digits summing to 9 must also get their a_i's turned to 0. Example:

$$A = 811765432113$$

$$B = 111234567897$$

$$\text{Target } A+X = 812000000000$$

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B — Big Brother

Problem

Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

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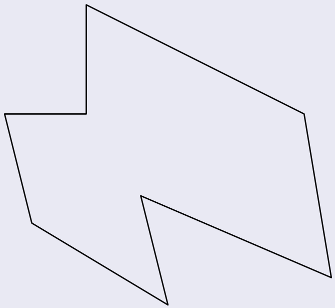
Solution



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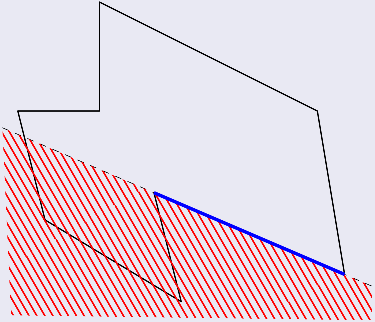
Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

Solution

Line segments of the polygon induce *half-planes*:
In order for points along that wall not to be obscured, we cannot be behind that wall.



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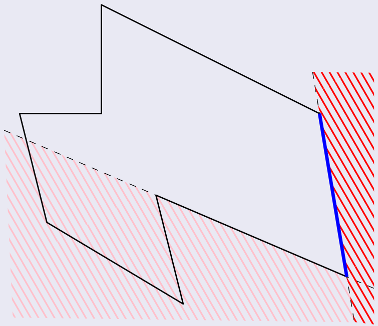

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Given polygon-shaped map of a room, find region from which all parts of the room can be seen.

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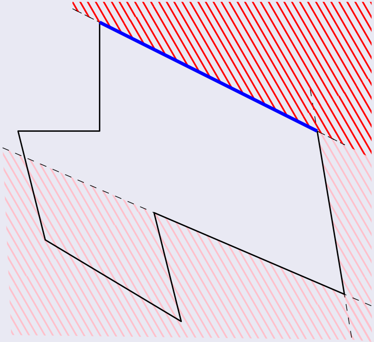

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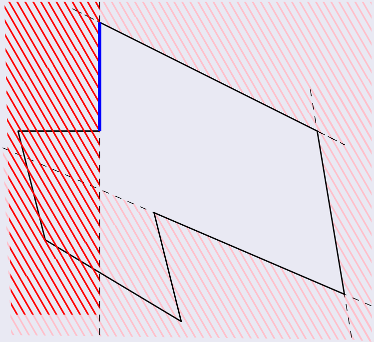

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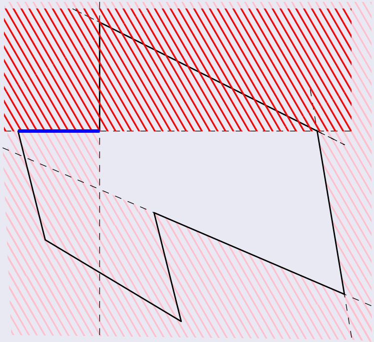

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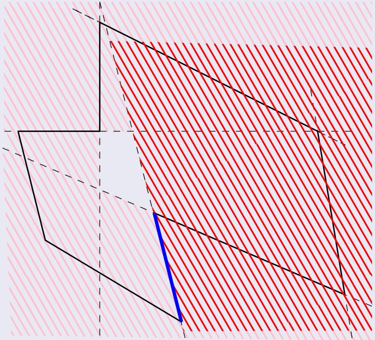

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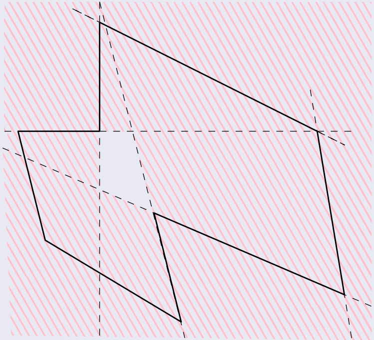

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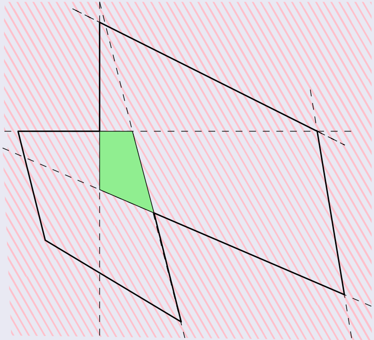
Seeing all parts of the room

$\Updownarrow$

Not “behind” any wall

$\Updownarrow$

Inside the intersection of the n half-planes.



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Statistics at 4-hour mark: 36 submissions, 11 accepted, first after 00:28

I — Infection Estimation

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Solution

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$$100, \quad 100 \cdot 1.01, \quad 100 \cdot 1.01^2, \quad \dots \quad 100 \cdot 1.01^i, \quad \dots \quad 5 \cdot 10^6$$

(around $\log_{1.01}(5 \cdot 10^6) \approx 1500$ different values)

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Statistics at 4-hour mark: 46 submissions, 6 accepted, first after 01:03

E — Exhaustive Experiment

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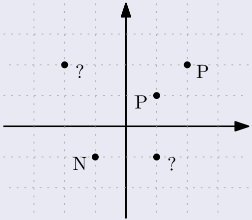
Solution (1/2)



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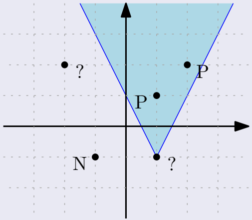

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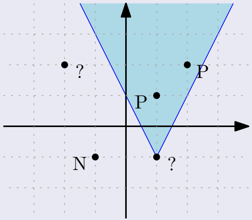

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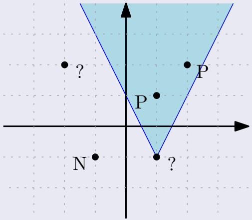

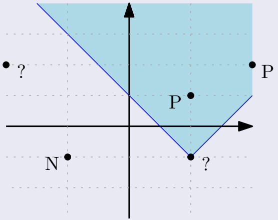

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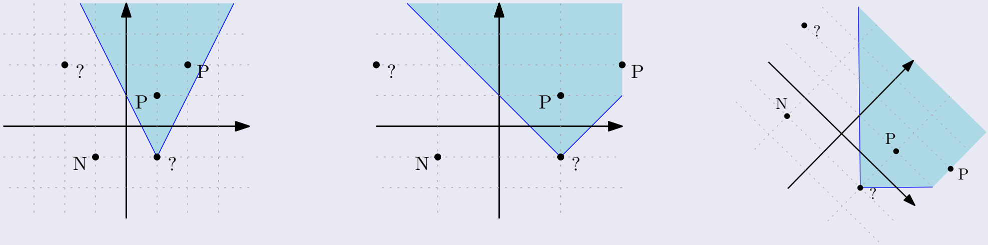

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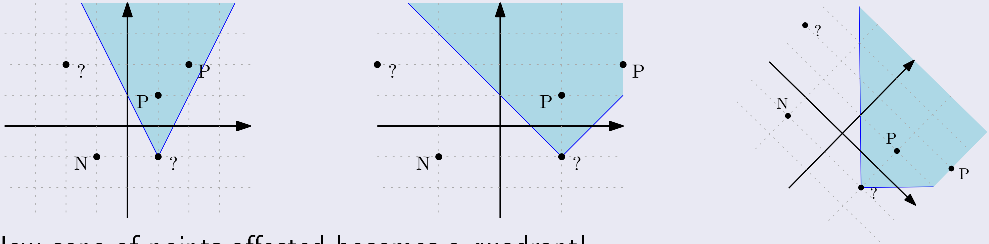
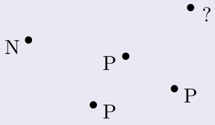


2. Now cone of points affected becomes a quadrant!

Solution (2/2)



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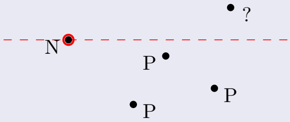

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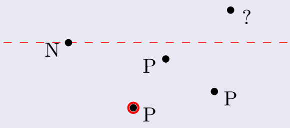

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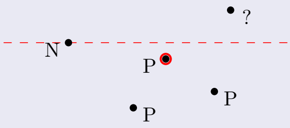

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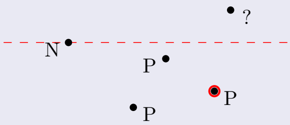

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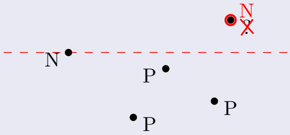

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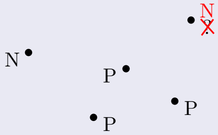

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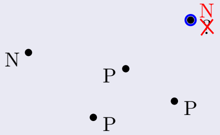

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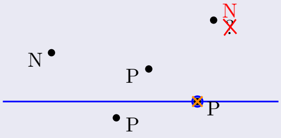

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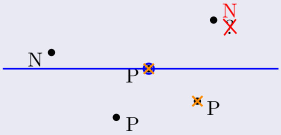

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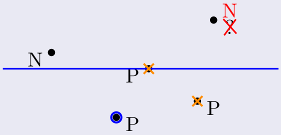

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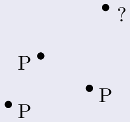

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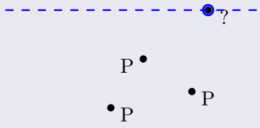

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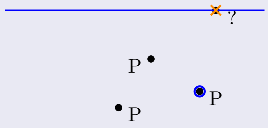

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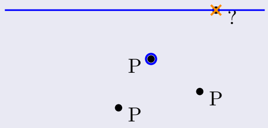

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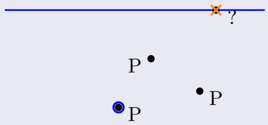

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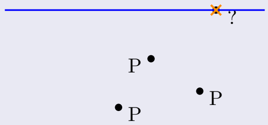

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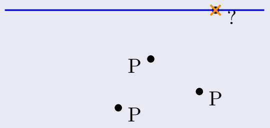
- Time complexity $O(n \log n)$ for sorting then $O(n)$.

E — Exhaustive Experiment

Solution (2/2)



- Negative tests: nothing in upper right quadrant of negative test can be faulty.
  1. Sweep from left to right, maintaining minimum y-coordinate of a negative point seen
  2. Points above current minimum must also be negative (if marked P $\Rightarrow$ "impossible")
- Greedily find smallest possible number of positive points:
  1. Sweep from right to left, maintaining maximum y-coordinate of a faulty point, and maximum y-coordinate of a *potentially faulty* point.
  2. When explanation for positive test needed, use best *potentially faulty* one seen so far.
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Statistics at 4-hour mark: 18 submissions, 3 accepted, first after 01:14

H — Hiring and Firing

Problem

A company hires and fires workers over up to 100 000 days. Assign an HR employee to each day so that for each worker a different HR employee is assigned the day they are hired and the day they are fired. Minimize number of HR employees used.

Solution (1/3)

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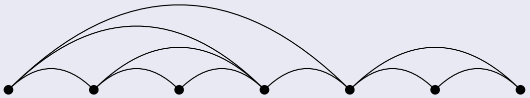
2. We seek the chromatic number of this graph.
3. Graph coloring is NP-hard in general and even in planar graphs.
   But these graphs are special... what do they look like?

Solution (2/3)

1. Mark the days on a timeline and draw an arc for each worker:



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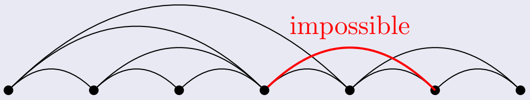

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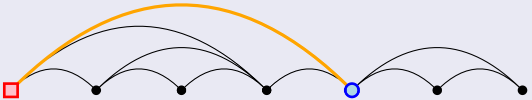

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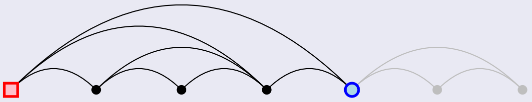

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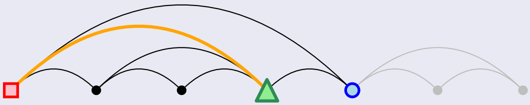

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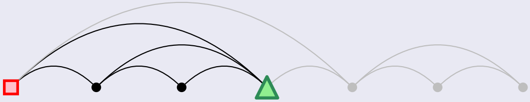

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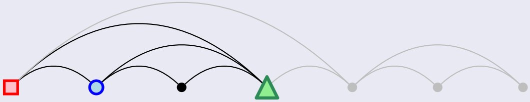

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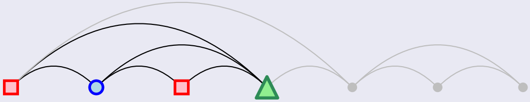

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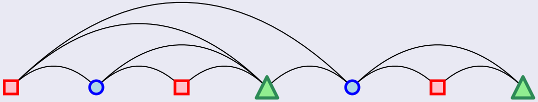

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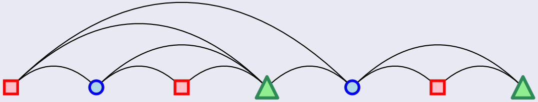

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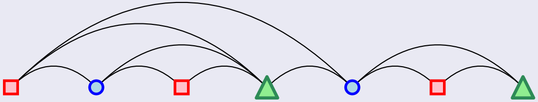

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5. Recurse on subproblems and repeat. Use a valid color for the middle vertex.
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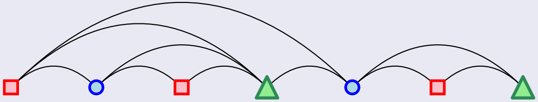

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   So if we have three colors, will always be a choice available for the middle vertex.
   In other words, 3 colors is always enough!

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1. Check if graph is 2-colorable (or 1-colorable), standard algorithm.

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Statistics at 4-hour mark: 22 submissions, 0 accepted, first after N/A

L — Language Survey

Problem

Find three connected regions in a grid. For each cell prescribed whether it should contain a single region or at least two different regions.

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Solution (1/2)

1. Try to partition grid into three *disjoint* connected regions with following property: for every cell (i, j) there is a neighboring cell (i', j') belonging to a different region.

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2. If we manage to find this, problem becomes easy:
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   - For every cell (i, j) where two or more regions should overlap, extend the region from (i', j') into (i, j).

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Solution (2/2)

1. How to find the starting point partition?

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Solution (2/2)

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ACCCCCCCC
ACBBBBBBB
ACBCCCCCB
ACBCCCBCB
ACBBBBBCB
ACCCCCCCB
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3. Case when dimensions are small is left as exercise...
   – when at least two rows and columns the above works
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Results!