

# NCPC 2017

## Presentation of solutions

The Jury

2017-10-07

## NCPC 2017 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Pål Grønås Drange (Statoil ASA)
- Markus Fanebust Dregi (Statoil ASA/Webstep)
- Antti Laaksonen (CSES)
- Ulf Lundström (Excillum)
- Jimmy Mårdell (Spotify)
- Lukáš Poláček (Google)
- Johan Sannemo (Google)
- Pehr Söderman (Kattis)

## J — Judging Moose

### Problem

Classify moose based on their horns.

### Some solution (guess the language)

```
solve(0, 0) :-  
    !, write('Not a moose').  
solve(L, R) :-  
    type(L, R, Type),  
    Val is 2*max(L, R),  
    write(Type), write(' '), write(Val).  
type(L, L, "Even") :- !.  
type(_, _, "Odd").
```

Statistics: 347 submissions, 252 accepted, first after 00:03

## B — Best Relay Team

### Problem

Pick best relay team, given runners' standing and flying start times.

### Solution

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Complexity is  $O(n \log n)$ . Many other solutions are also possible.

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Complexity is  $O(n \log n)$ . Many other solutions are also possible.

Statistics: 491 submissions, 189 accepted, first after 00:08

# G — Galactic Collegiate Programming Contest

## Problem

There are  $n$  teams who solve  $m$  problems in an ICPC style programming contest. After each successful submission, print the rank of your team.

## Solution

- 1 Maintain a set  $S$ : the teams whose score is better than your team's score. Your rank is  $|S| + 1$ .

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Statistics: 578 submissions, 79 accepted, first after 00:29

# I — Import Spaghetti

## Problem

The dependencies form a directed graph, and the task is to find a shortest cycle in a directed graph.

## Solution

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Statistics: 290 submissions, 52 accepted, first after 00:30

# E — Emptying the Baltic

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Statistics: 296 submissions, 55 accepted, first after 00:47

## D — Distinctive Character

### Problem

Given  $n$  bit vectors of length  $k$ , find a bit vector whose minimum Hamming distance is maximum.

### Solution

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Statistics: 461 submissions, 40 accepted, first after 00:17

## C — Compass Card Sales

### Problem

Dynamically keep track of “uniqueness values” of cards while cards are being sold off.

### Solution

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Dynamically keep track of “uniqueness values” of cards while cards are being sold off.

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- 1 When card is sold, at most 6 other cards (the 2 “adjacent” cards of each color) can change their uniqueness values.

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Complexity is  $O(n \log n)$  with balanced search trees or similar.

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Statistics: 105 submissions, 14 accepted, first after 01:10

### Problem

Assign pool of weak/normal/strong people to 2-person kayaks (of different speed factors) to maximize speed of slowest kayak.

### Solution

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## Solution

- 1 Binary search over the answer.
- 2 Check feasibility by greedily assigning people to kayaks
  - kayaks requiring strong+strong or strong+normal get that
  - kayaks that can handle weak+weak or weak+normal get that
  - pair up remaining weaks with strongs and normals with normals and check if this can make all kayaks fast enough

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Time complexity is  $O(n \log n)$  for  $n$  people.

Statistics: 82 submissions, 23 accepted, first after 00:46

## A — Airport Coffee

### Problem

Find fastest way of walking distance  $L$ . At certain points  $x_i$  we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

### Solution

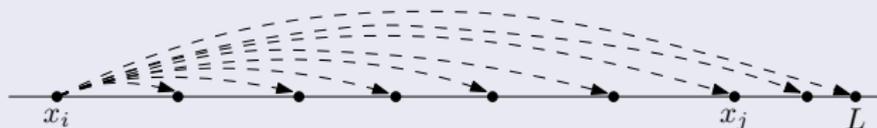
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- 1 Let  $S(i)$  be best time if we start from  $i$ 'th cart.
- 2 Easy dynamic programming: for each  $j > i$ , try buying next coffee at cart  $j$

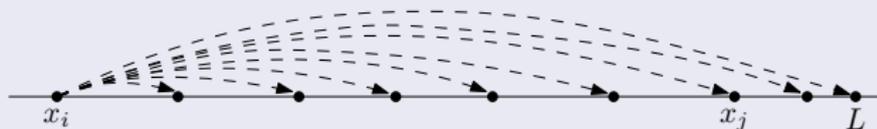
$$S(i) = \min_{j>i} S(j) + \text{Time to go from } x_i \text{ to } x_j$$

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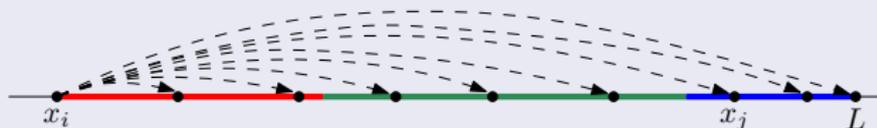
- 3 Alas, leads to  $\Omega(n^2)$  time – too slow!.

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(get next coffee as soon as possible)

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During cooldown, during drinking, and after finishing the coffee
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(get next coffee as soon as possible)
- 3 During drinking: best to pick **largest** such  $j$   
(keep drinking coffee as long as possible)

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Statistics: 65 submissions, 7 accepted, first after 02:50

## Problem

Given a set of rays from in 2D  $(0,0)$ , and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

## Phase 1

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- 4 Can also check this using only integer computations. (But, despite small coordinates, need 64 bits.)
- 5 Points with a unique neighboring ray can be immediately assigned to that ray (if it has capacity left).

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Given a set of rays from in 2D  $(0,0)$ , and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

## Phase 2

- 1 For remaining points (having two closest rays) we get a bipartite matching problem between points and rays.

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  - 1 Time complexity with Ford-Fulkerson is  $O(p^2)$  where  $p$  is the number of train lines adjacent to some remaining person.
  - 2 However  $p$  is hard to analyze. Turns out that  $p \approx \text{max coordinate} = 1000$ , so this approach is fast enough.

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- 3 Approach 2: greedyish solution

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- 1 For remaining points (having two closest rays) we get a bipartite matching problem between points and rays.
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Statistics: 17 submissions, 0 accepted

# F — Fractal Tree

## Problem

Given a huge tree with potentially  $100000 \cdot 2^{30}$  vertices, find distances between pairs of vertices.

## Solution

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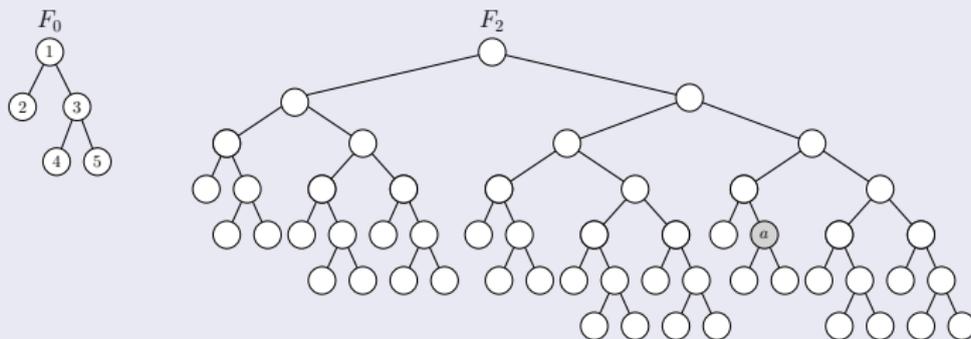
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- 3 Reduces the problem to  $k \leq 30$ .

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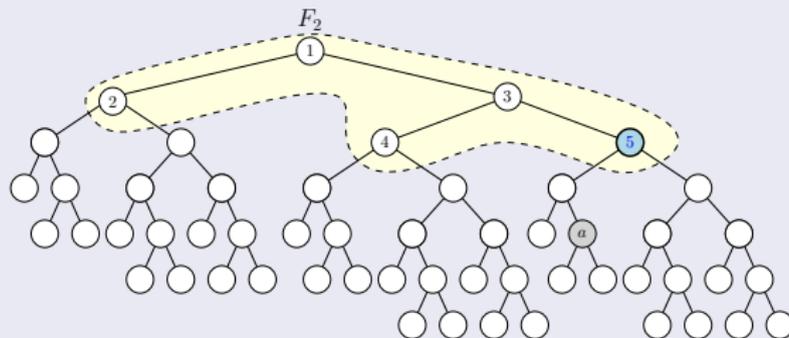
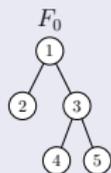
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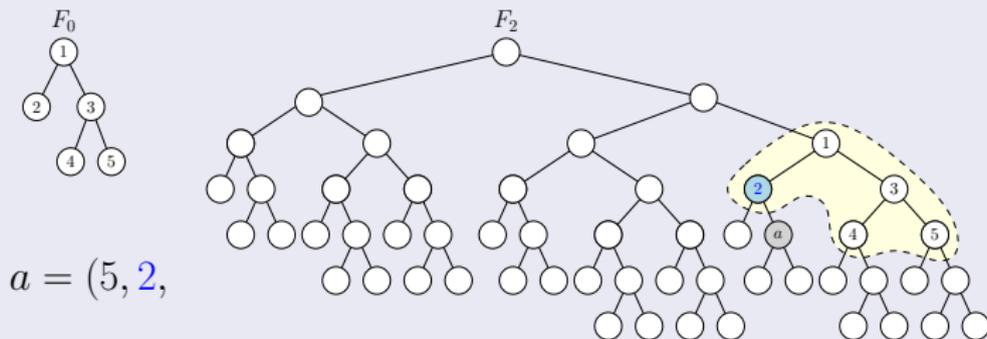
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$$a = (5, 2,$$

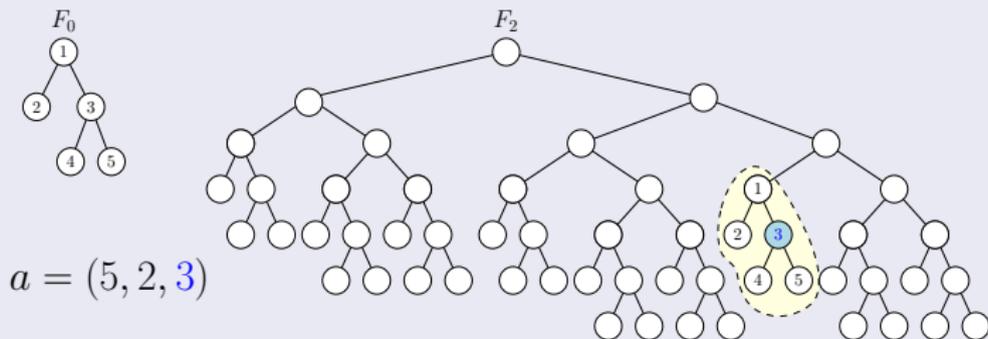
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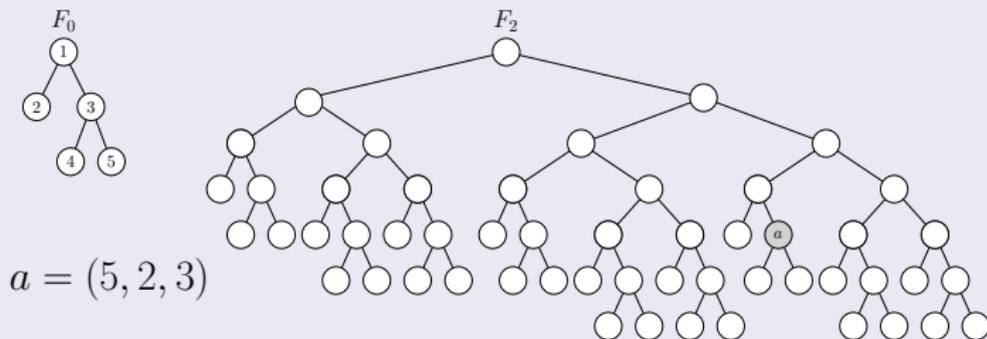
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  - 2 Finally add which node  $a$  corresponds to in last copy of  $F_0$ .

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Given a huge tree with potentially  $100000^{2^{30}}$  vertices, find distances between pairs of vertices.

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- 1 Representation of vertex  $a$  in the big tree.
- 2 Can find this representation in  $O(k \log n)$  time using binary search and precomputation of subtree sizes.

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Statistics: 5 submissions, 0 accepted

253 teams

611 contestants

2819 total number of submissions

10 programming languages used by teams

Ordered by #submissions: C++ (1016), Java (865), Python (763), C (67), C# (65), Haskell (16), Prolog (9), Scala (8), Go (6), Ruby (4)

438 number of lines of code used in total by the shortest **jury** solutions to solve the entire problem set.  
(Significantly smaller than previous years – no killer problem in terms of implementation this year.)

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- The jury wrote Python solutions for all problems except C (**Compass Card Sales**). But mostly in Python 2, which is faster than Python 3 on Kattis due to using pypy instead of CPython. The Python solutions are always the shortest (often by a wide margin).

# What now?

- Northwestern Europe Regional Contest (NWERC):  
November 26 in Bath (UK).  
Teams from Nordic, Benelux, Germany, UK, Ireland.



- Each university sends up to two teams to NWERC to fight for spot in World Finals (April 2018, in Beijing, China)