NCPC 2013
Presentation of solutions

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NCPC Jury

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G – Erase Securely

Problem

Given binary hard disk content before and after described "safe" file removal procedure, decide whether it was successful.

Solution

- **Insight:** Each time the file is overwritten, all of its bits are flipped
- Odd-th number of overwrite: flips all bits once
- Even-th number of overwrite: Undoes previous flipping
- In case of odd number of total overwrites, all bits must appear as if flipped once for the removal to be considered "successful". Otherwise, both binary strings must be identical.

A – Planting Trees

Problem

Find an optimal planting schedule to minimize the earliest date when all the trees are mature.

Insight and Solution

- **Insight:** It is optimal to plant the trees in descending order of growth times.
  - **Proof**: If any two trees are not in this order, then swapping them cannot increase the result.
- **Solution:** Sort the trees by decreasing growth times, and output $\max(\text{position} + \text{growth time} + 1)$.
- **Time complexity:** $O(n \log n)$.

F – Timebomb (1/2)

Problem

Translate ASCII digits into memory variables

Insight and Solution

- **Insight:** The code is not known until the input has been read completely.
- Store the ASCII representation in memory, e.g., matrix of char and compute the number of digits from the string length.
- Translate the code into a number by using the appropiate power of 10 for each digit and compute the sum of them.
  - Stop if we find an invalid digit.
- Check if the number is zero modulo 6.
- But, what if the problem would have allowed 100-digit codes?

F – Timebomb (2/2)

Alternative solution

- Compute the **sum** of the digits.
- Use the fact that a number is divisible by 6 if it is an even number and the sum of its digits is divisible by 3.

E – Virus Replication

Problem

Find the minimum substring that needs to be changed to turn one string into another.

ACCGTCA
ACCTTGCA

Insight

The beginning and end of the two strings should be identical up until the change.

Solution

- First difference from start of strings marks start of change.
- First difference from end of strings marks end of change.
- Check that the start of the change is before its end in both strings.

D – Robert Hood

Problem

Find the longest distance between any pair of points in a set.

Insights and Solution

- **Insight:** The interesting points are those in the convex hull of the set.
- Find the convex hull using a known fast algorithm.
- **Insight:** The small integer coordinate range $[-1000, 1000]$ means that the number of points on the convex hull will be reasonably small.
- An algorithm with N^2 complexity is sufficiently fast to find the longest distance, i.e. you can examine all possible pairs of points on the hull.

B – Boiling Vegetables

Problem

Find the minimum number of cuts of vegetables, such that the smallest and largest piece are not very different.

Insights and Algorithm

- **Insight:** All cut pieces of one vegetable are of the same weight.
- Loop over all possible candidates for the largest weight piece.
- For each vegetable piece, calculate how many cuts are needed to get all resulting pieces weigh at most the assumed maximum. Check if they are heavy enough to pass the ratio T from the input.
- **Optimizations:** Go to a next candidate for the largest piece if you already cut too many pieces. Complexity is $500N \log N$.

J – Dartboard

Problem

Calculate the expected score of throwing one dart.

Insights and Solution

- **Insight:** The problem is radially symmetric after you average the numbers of all pies.
- **Insight:** The integral over the radial variable r can be calculated explicitly.
- The final result can be calculated in constant time as a sum of the scores over six different radius intervals.
- Alternatively, a sufficiently efficient numerical integration scheme can also be used.

C – Number Trick (1/2)

Problem

Given X find numbers A for which moving the first digit to the end of the number is equivalent of multiplying by X.

Insight

Assume A has n digits and starts with the digit A_0. Moving the first digit to the end then turns it into

$$A' = (A - 10^n A_0) \times 10 + A_0,$$

which should equal $A \cdot X$, so

$$A \cdot X = 10A - (10^{n+1} - 1)A_0 \quad \Rightarrow \quad A = \frac{10^{n+1} - 1}{10 - X} A_0$$

C – Number Trick (2/2)

Solution

Loop over the number of digits n and all starting digits A_0 to check whether the A obtained from

$$A = \frac{10^{n+1} - 1}{10 - X} A_0$$

is a valid solution.

K – Cliff Walk

Problem

Find the furthest reachable point while staying clear of the tide.

Insights and Algorithm

- Find the earliest entry and latest exit time for all squares using the provided formula.
- Run shortest path from home at $t = 0$ to all positions.
- Run shortest path from home at $t = 12$ to all positions, but backwards in time.
- Use priority queue to make algorithm run fast.
- **Insight:** go through the map and see where arrival time is earlier than departure time.
- Return the maximum of the distances to these squares.

I – Dance Reconstruction (1/2)

Problem

Given a permutation, find its kth root.

Permutation Exponentiation

- **Example:** Take permutation to the 20th power
  - A cycle of length 20 breaks down into 20 cycles of length 1
  - Length 30: 10×3
  - Length 40: 20×2
  - Number of cycles: greatest common divisor of 20 and length
- A solution should merge small cycles back into big ones

I – Dance Reconstruction (2/2)

Algorithm

- Decompose input into cycles
- **Insight:** When merging m cycles of length ℓ into one cycle of length $m \cdot \ell$, $\gcd(m \cdot \ell, k)$ must be m
- **Example:** If $k = 20 = 2^2 \cdot 5$ and $\ell = 2$, m must be a multiple of 4
- If $p^q \mid k$ and $p \mid \ell$, m must be a multiple of p^q.
- $m = \gcd(\ell, k)$ gives wrong answer
- Carefully place elements on the big cycle

H – Pinball

Problem

Simulate ball falling down a pinball board

Solution

- Sweep board as ball falls
- Event queue, sorted by y
  - Events: Add/erase segment, possible ball hit
- Search tree with segments, sorted by x at current y
  - Query: which segment would the ball hit?
  - Update: add/erase segment
- **Insight:** No intersections $\Longrightarrow$ order in the tree independent of y