German Collegiate Programming Contest

GCPC Jury

gcpc@nwerc.eu

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jury sample solutions

Problem	**min. LOC**	**max. LOC**
Faculty Dividing Powers	31	54
Genetic Fraud	32	48
Indiana Jones and the lost Soccer Cup	50	60
Magic Star	49	74
Magical Crafting	46	93
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Sightseeing	34	113
Suiting Weavers	88	179
Time to live	32	65
$\sum$	460	932

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- So solution is $\min_{i \geq 1, k_i > 0} \left\{\lfloor n_i / k_i \rfloor\right\}$

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- Number of prime factors is $O(\log k)$, evaluating the formula takes time $O(\log n)$ for each one
- So whole runtime is $O(\sqrt{k} + \log k \cdot \log n)$
- Mistakes:
  - Scanner.nextInt()
  - while(ktmp¡=n) vs. while(ktmp¡=n/primes[m].first)

Faculty Dividing Powers

- Example: $n = 15$, $k = 12$
- $k = 12 = 2^2 3^1$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2^1		x		x		x		x		x		x		x	
2^2				x				x				x			
2^3								x							
3^1			x			x			x			x			x
3^2									x						

- $n!$ contains $2^{11}3^6$
- $i = min(\frac{11}{2}, \frac{6}{1}) = 5$

Genetic Fraud

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- Die Frage ist ob eine Subsequenz der Länge L/2 oder r existiert, bei der leichte Fehler (+/- 1) erlaubt sind
- Zur Lösung reicht das Standardverfahren für LCS (dynamische Programmierung) ebenfalls aus
- Problematisch konnte hauptsächlich Division durch zwei werden
- durch fehlerhaften Testinput: Greedy wurde auch akzeptiert

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- If all nodes have been removed, but at one point there was more than one node with indegree 0, output "`missing hints`"
- Else output nodes in the order they were removed
- common mistake: stopped immediately once more than one node with indegree 0 was available, instead of checking whether all nodes will be removed

Magic Star

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- The real number of possible solutions is considerably smaller.
- Construct all possible solutions and select the lexicographically smallest one.
- Better: construct the solutions in lexicographical order, then you can stop as soon as the first solution is found.

Magical Crafting

- Gegeben ist ein Set von Rezepten und ein Leuchteffekt
- Herauszufinden ist, ob ein Leuchteffekt aus den Rezepten erstellt werden kann . . .
- . . . und wenn ja, mit welchen Kosten

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- Die Rezepte sind Regeln einer Grammatik in (beinahe) Chomsky Normalform
- Der CYK Algorithmus berechnet die Möglichkeit
- Erweitert um den Kostenfaktor liefert er auch die minimale Anzahl Diamanten

My brother's diary

- No-Brainer
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- is the most frequent letter unique?
- shift distance d is calculated by $(26 + (maxChar - \text{'E'}))\%26$

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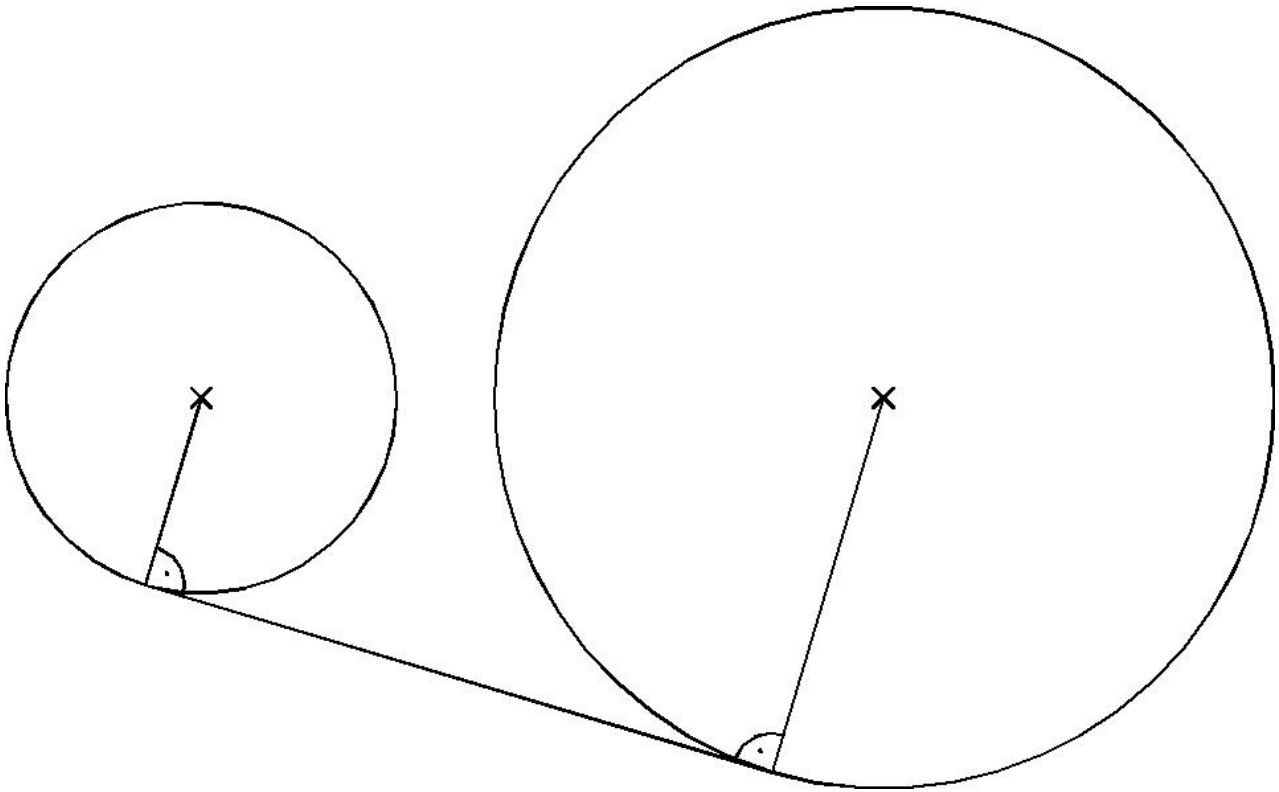
- count letter frequencies
- is the most frequent letter unique?
- shift distance d is calculated by $(26 + (maxChar - 'E'))\%26$
- calculated correct distance d, but not in interval $0 \leq d \leq 25$

Security Zone

- reduction to two circles:



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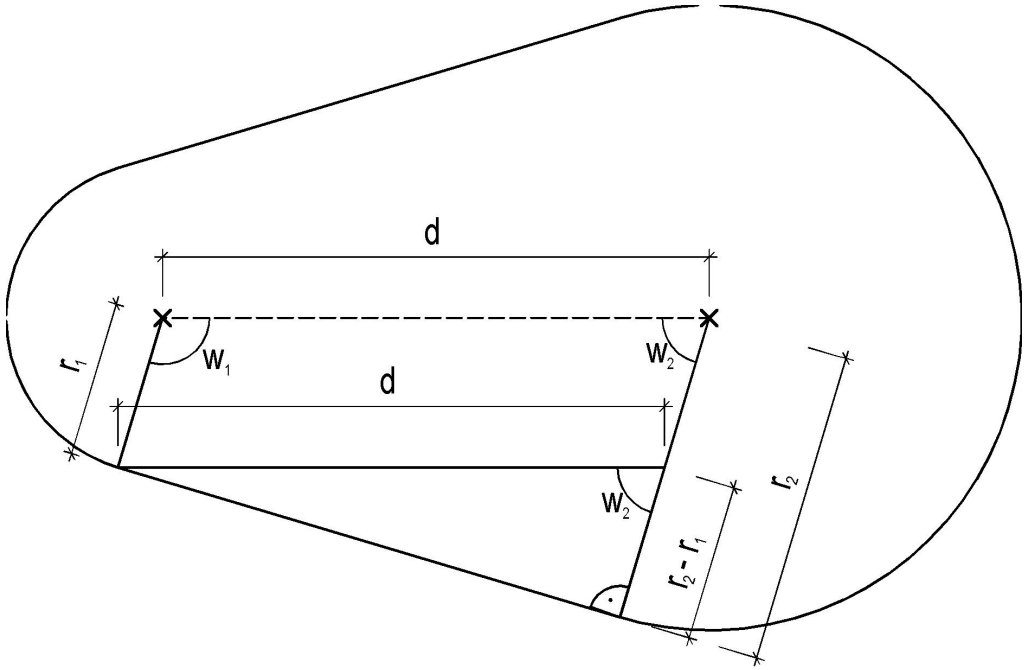

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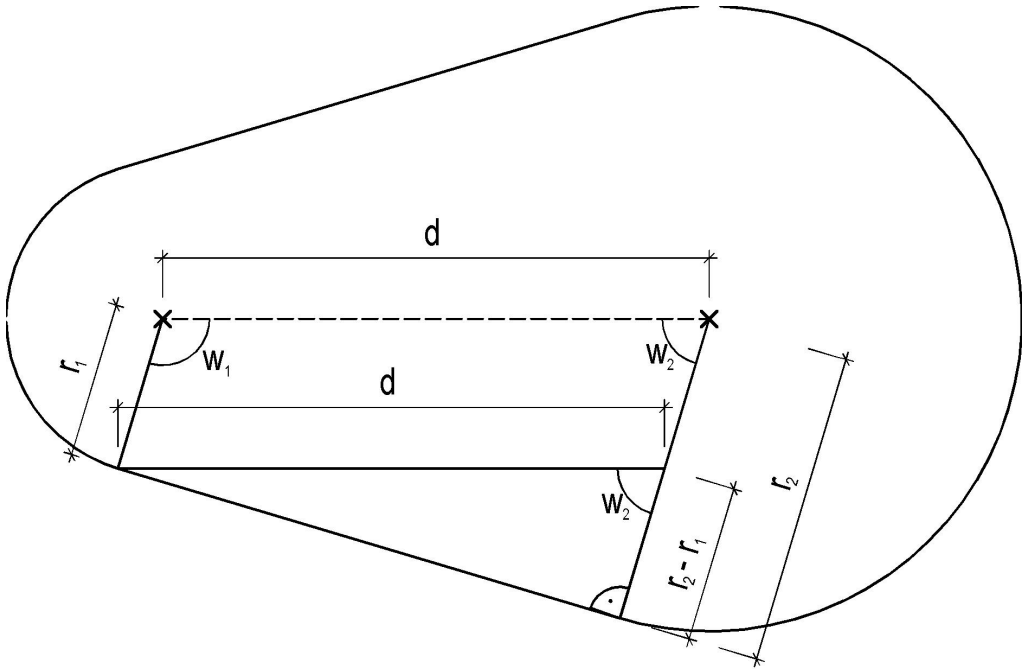

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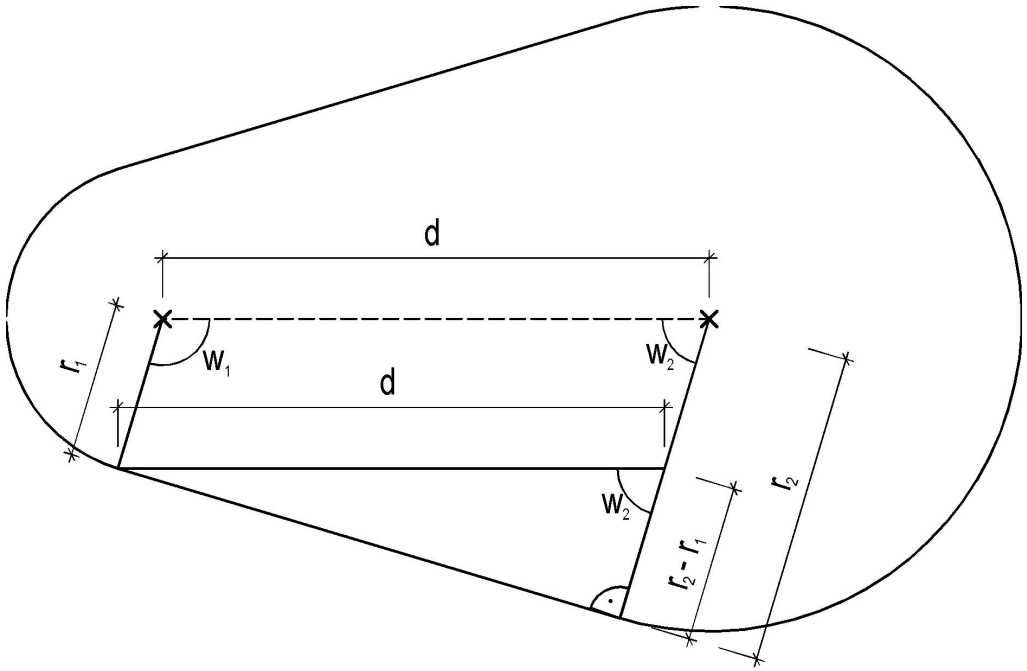

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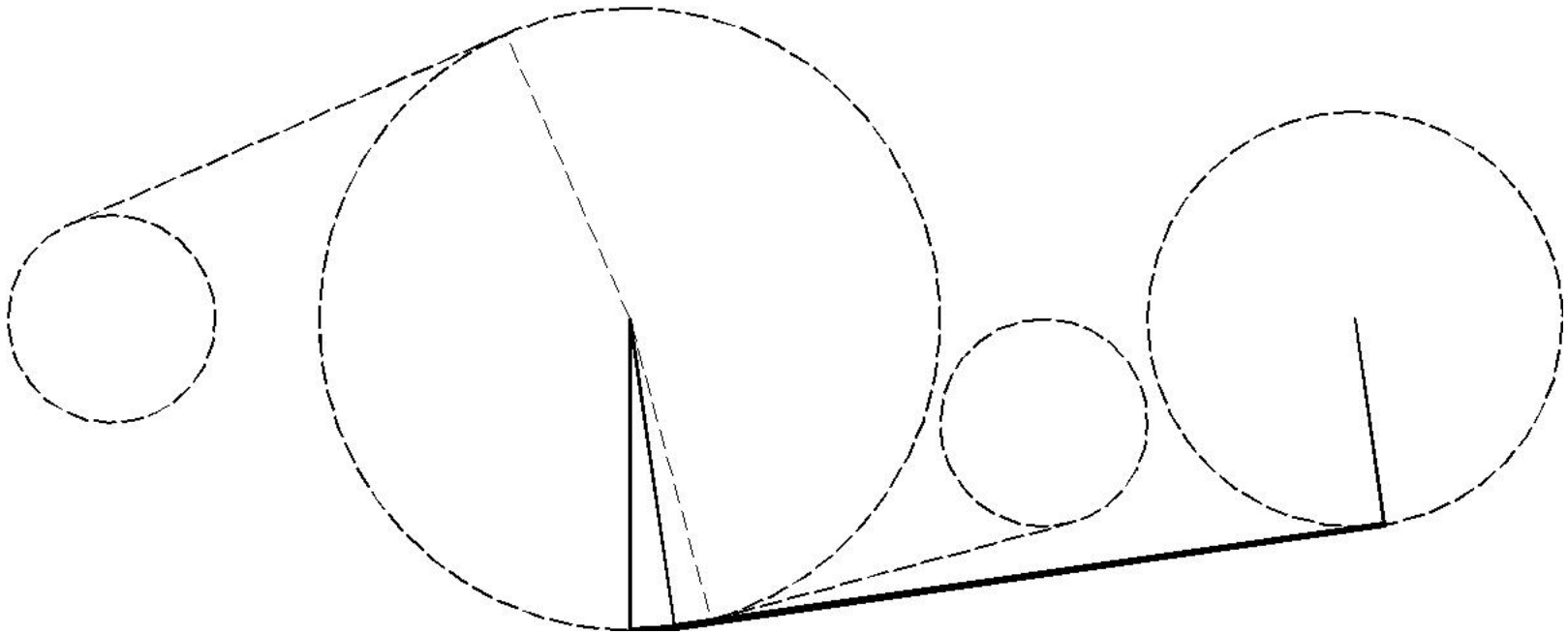
  - add radial part between last angle and next angle
  - add distance between the two points on the corresponding two circles

Security Zone

- example:



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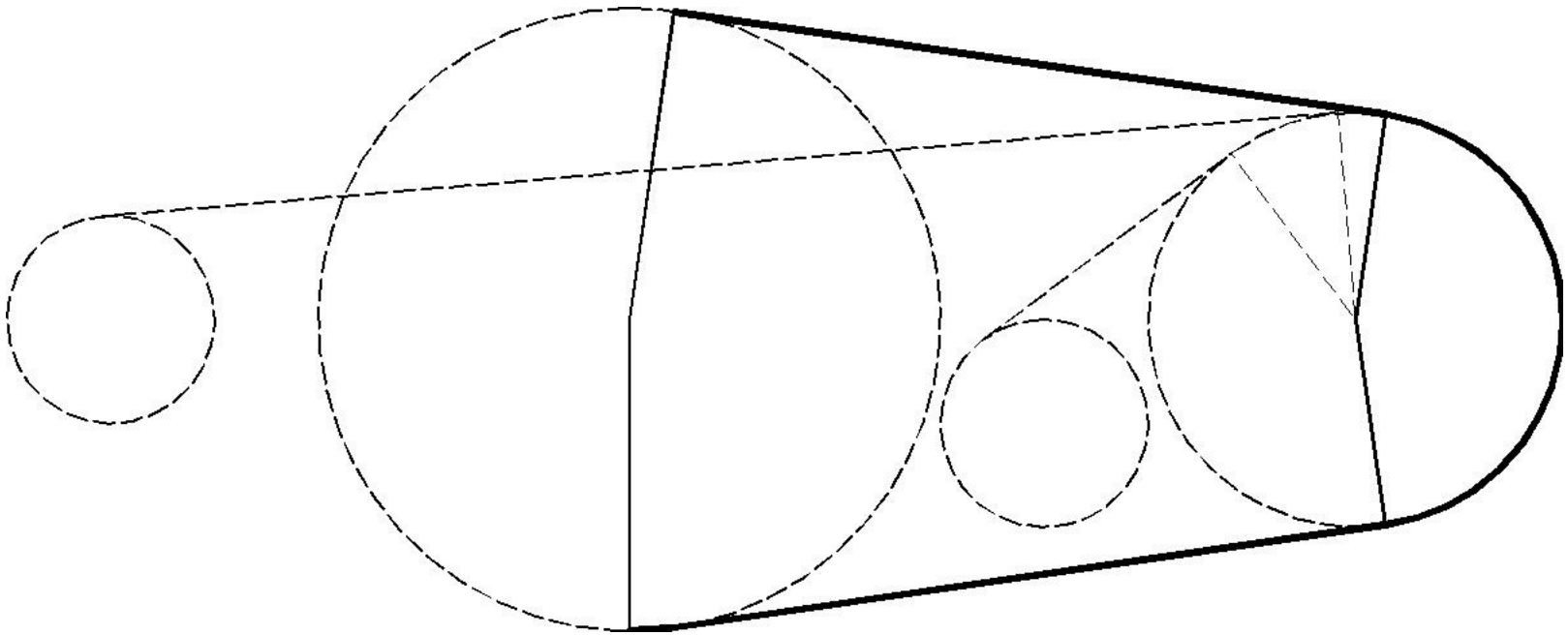

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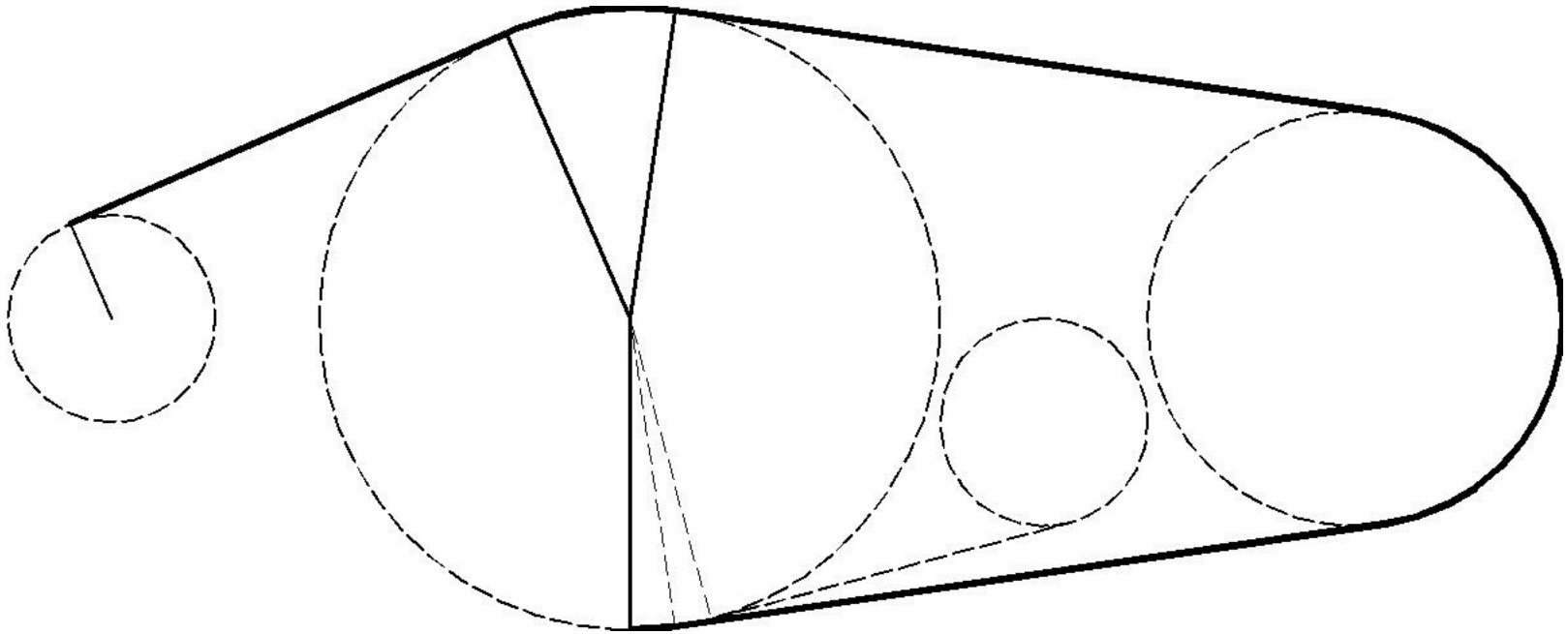

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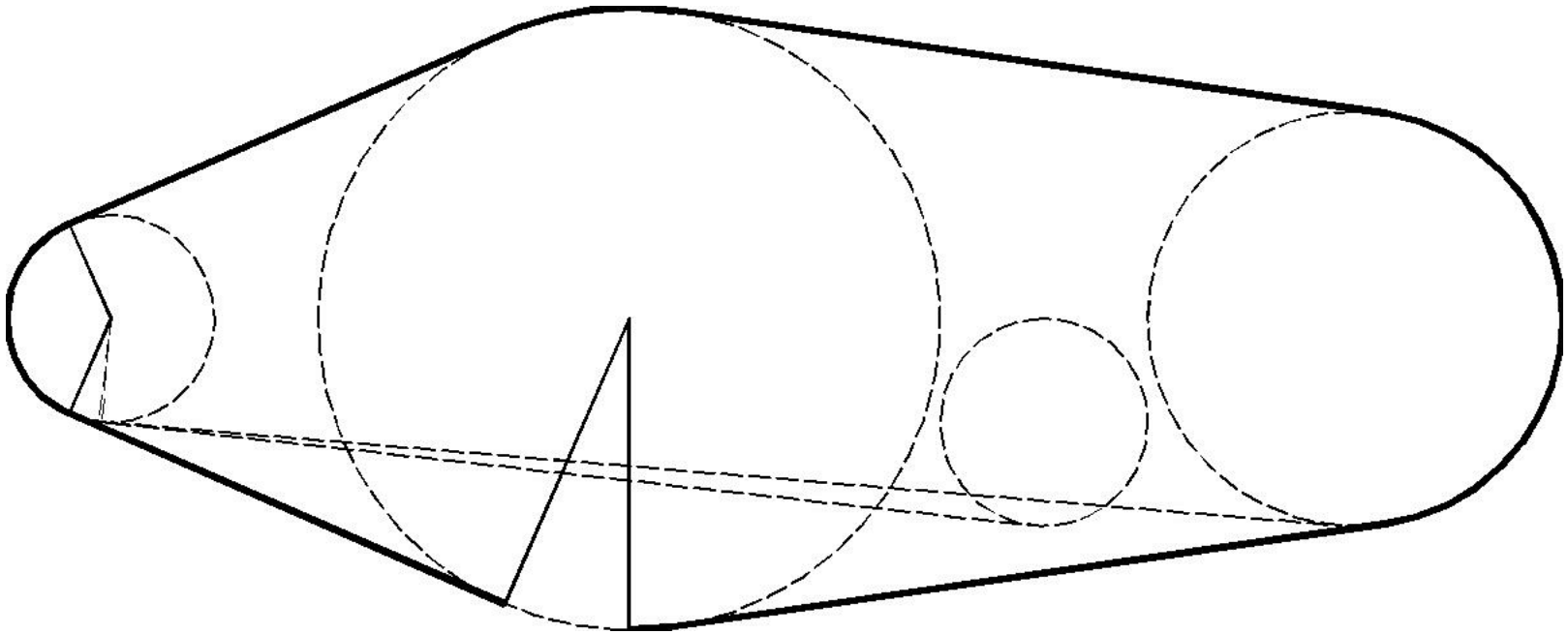

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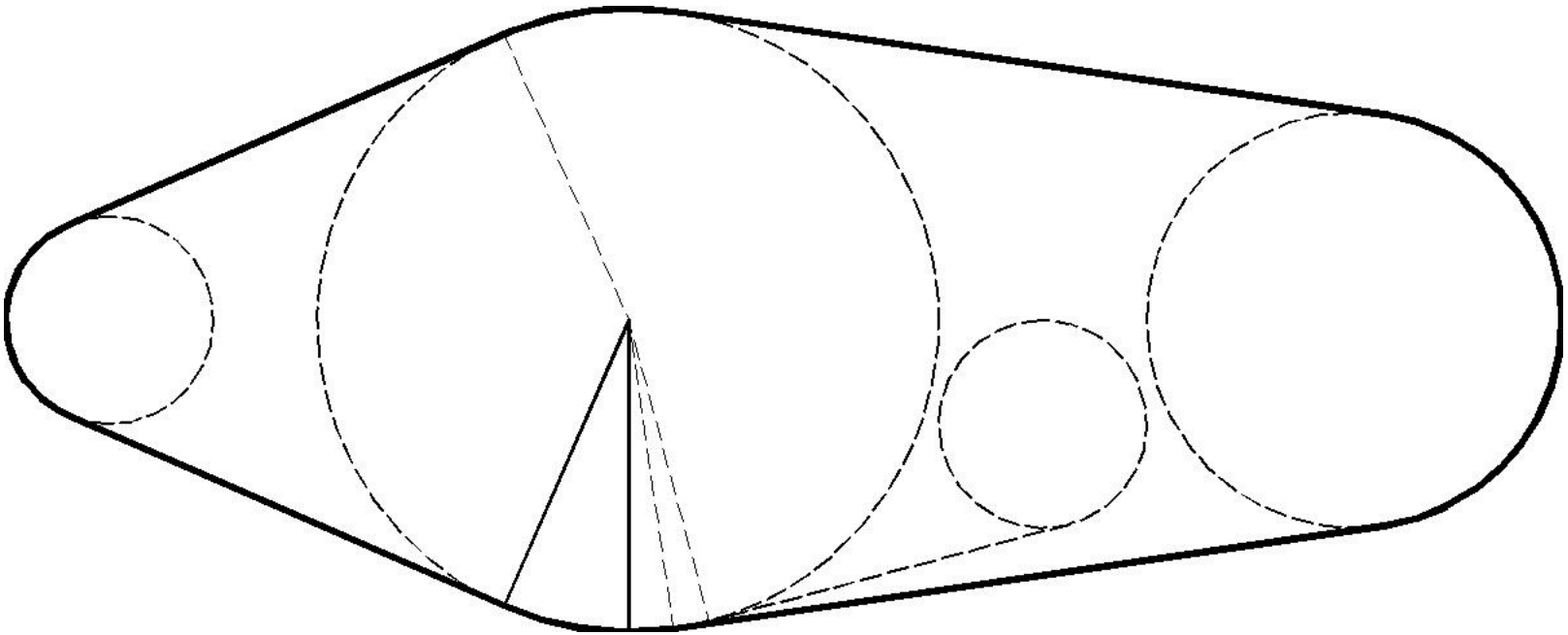

Security Zone

- common mistake: you did not submit

Sightseeing

- Gesucht ist eine kürzeste Route über ein zirkuläres Set von Strecken
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- Keine der Strecken darf dabei mehrfach durchlaufen werden
- Bei bekanntem Startpunkt lässt sich der kürzeste Weg in Linearzeit berechnen (Simple Graphstruktur)
- $\Rightarrow$ Simulation durch Dynamische Programmierung
- Die Strecke musste vom Ziel her rekonstruiert werden
- Auch ausreichend: Dijkstra Algorithmus
- Integer Überläufe konnten richtige Zeit voräuschen

Suiting Weavers

Problem

Possible to assign fibers to weavers, so that no weaver has more fibers than Willy?

General Approach

- Willy picks up all fibers he can reach
- Calculate maximum fibers that each weaver may pick up not surpassing Willy
- Use maximum flow algorithm to decide

Suiting Weavers (2)

Assume Willy collects all fibers in reach

- Find these places
- Assign fibers to Willy
- This gives Willy's optimum number of fibers W

Shortcut: If there exists a weaver with more than W fibers $\Rightarrow$ `Lonesome Willy`

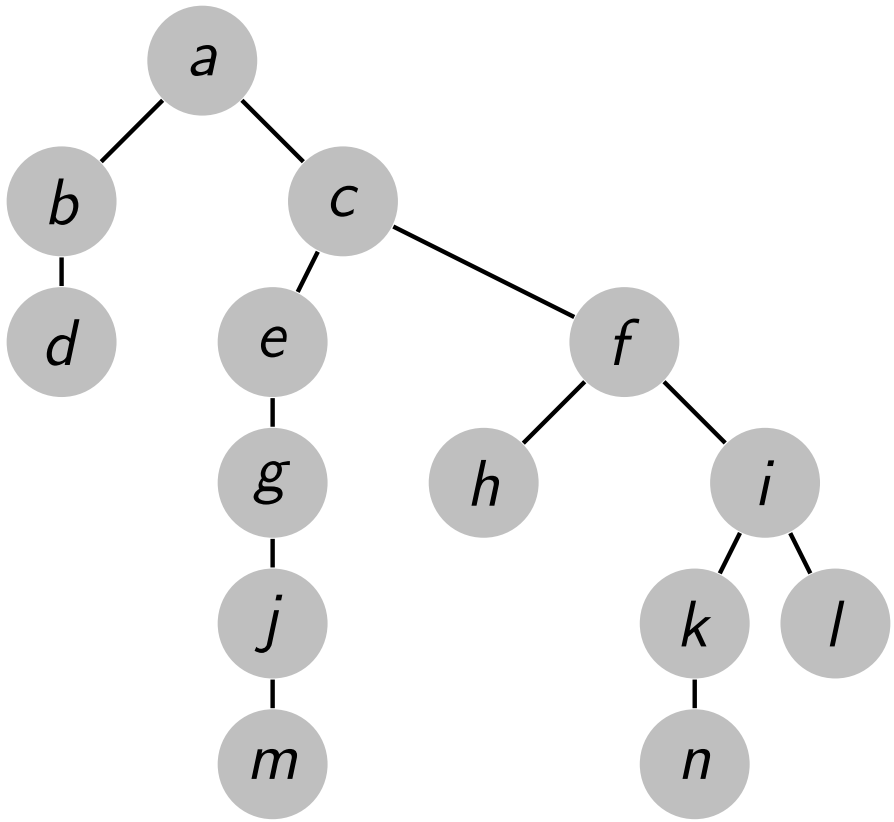
Suiting Weavers (3)

Maximum flow network

- One node for each remaining place and weaver, an additional source and a sink
- Edges
  - From each place to weavers that can reach it
    Capacity: number of fibers (or ∞)
  - From source to each place
    Capacity: number of fibers of the place
  - From all weavers to sink
    Capacity: difference of W and the initial number of fibers collected by the corresponding weaver
- max. flow $= \sum f_i \quad \Leftrightarrow \quad$ `Suiting Success`
  i.e., all remaining fibers were assigned

Time To Live

- given: tree
- find: node X (resp. path length) – the path length from any node in the tree to X should be minimized



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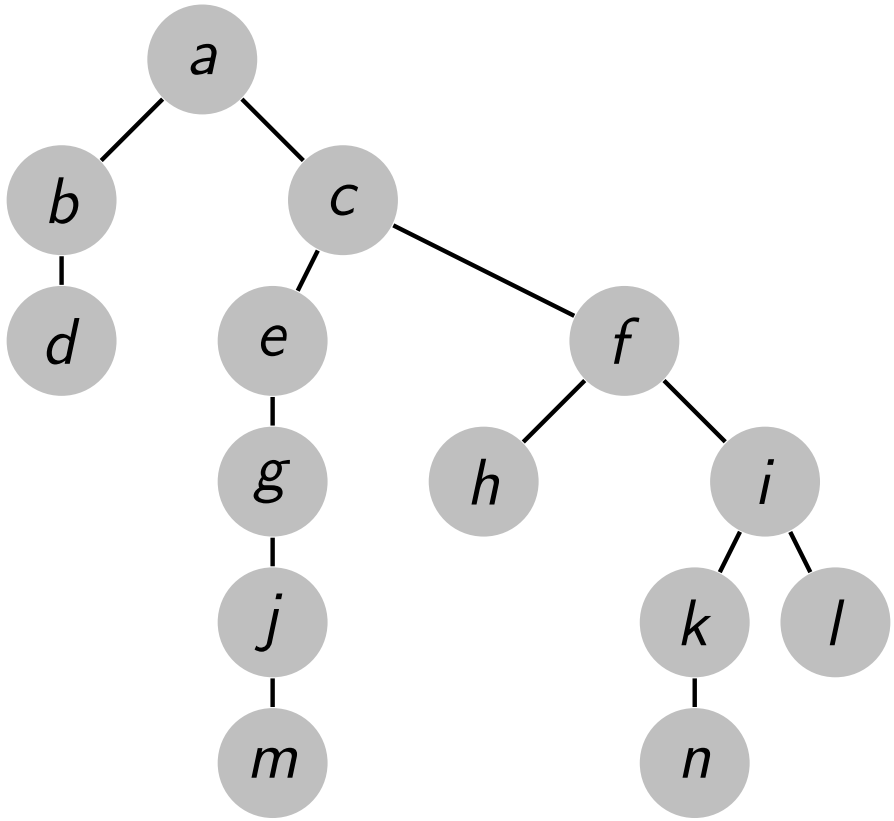
- $\Rightarrow$ compute length l of longest path in tree, answer: $(l+1)/2$
- do BFS from arbitrary root first, then do BFS from last node in previous BFS



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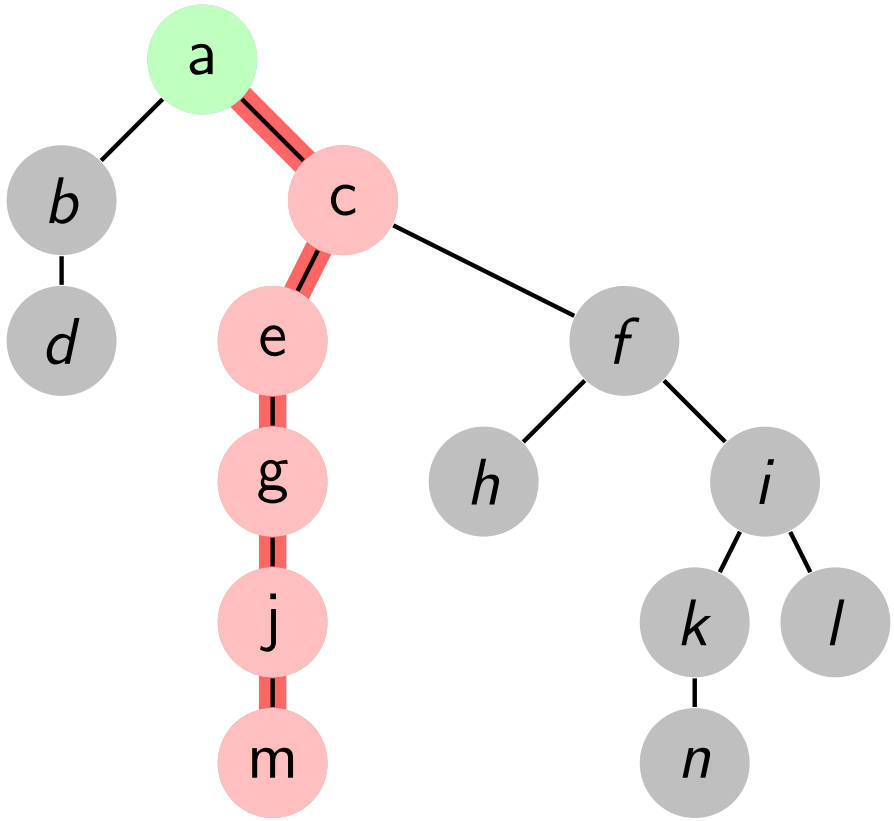

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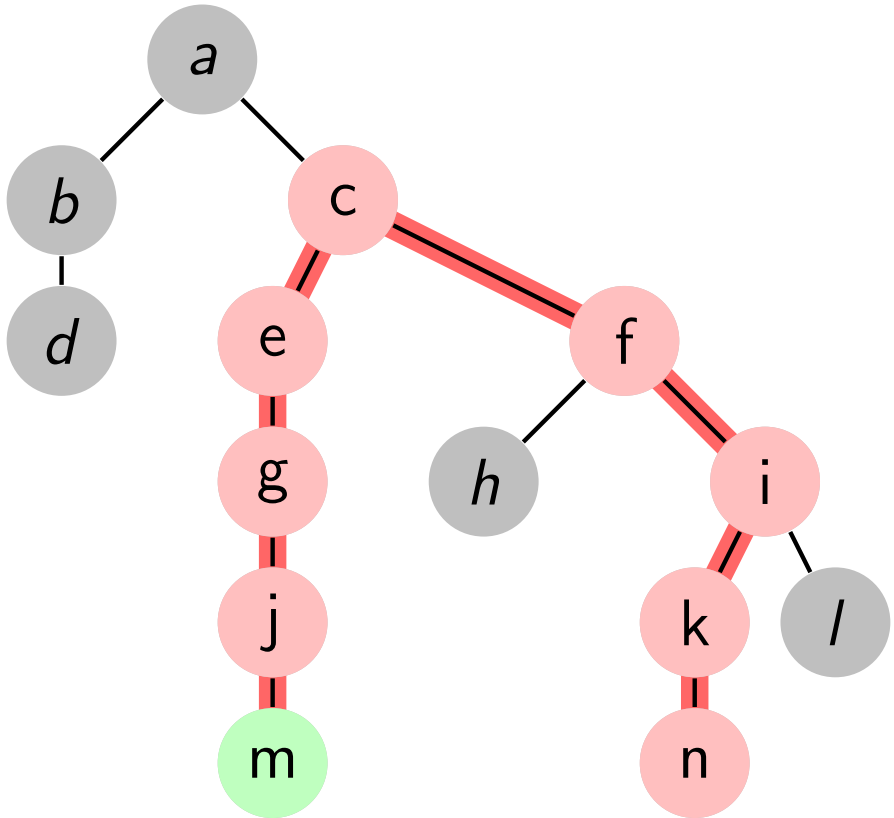
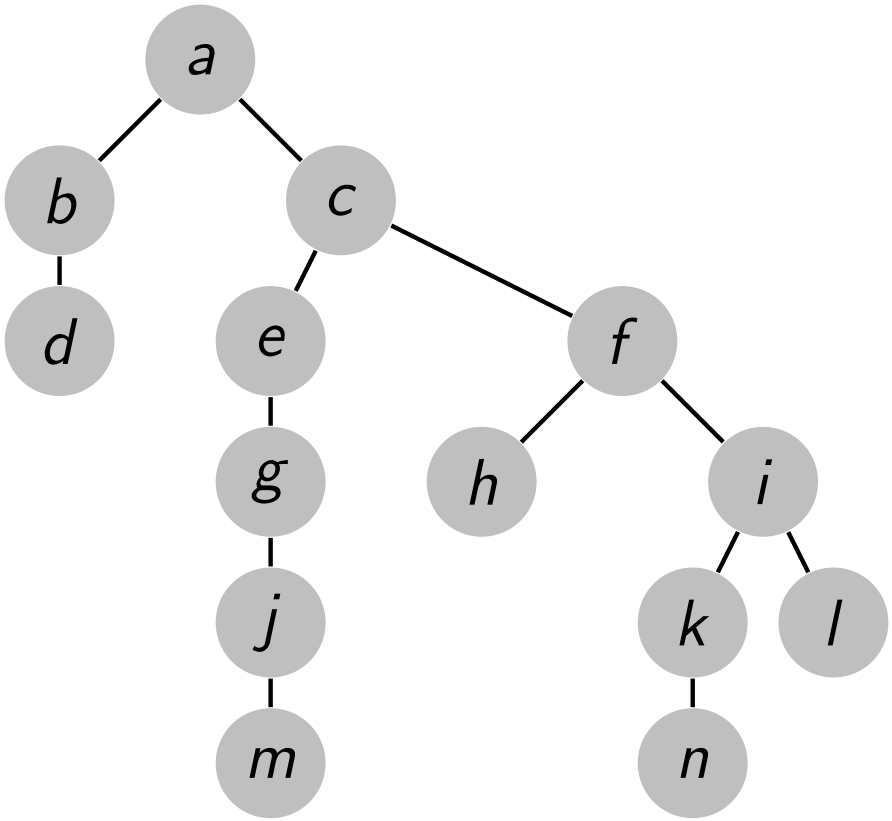


Time To Live

- alternative: number „layers“ in tree
- leafs on layer 0, other nodes on layer $1 + max(successors)$



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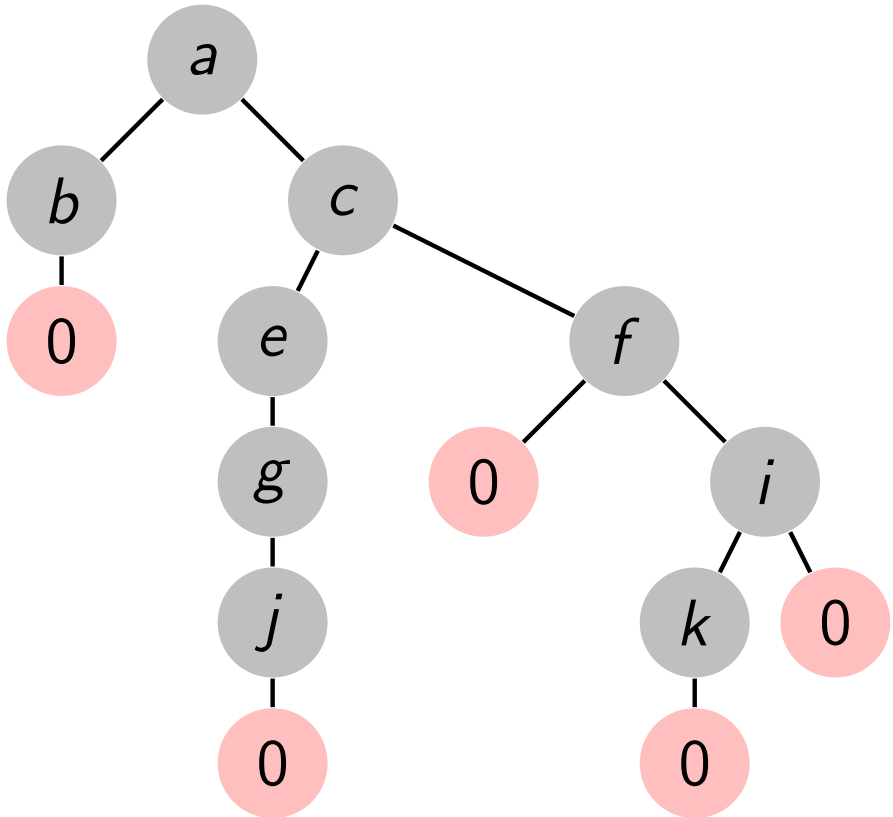

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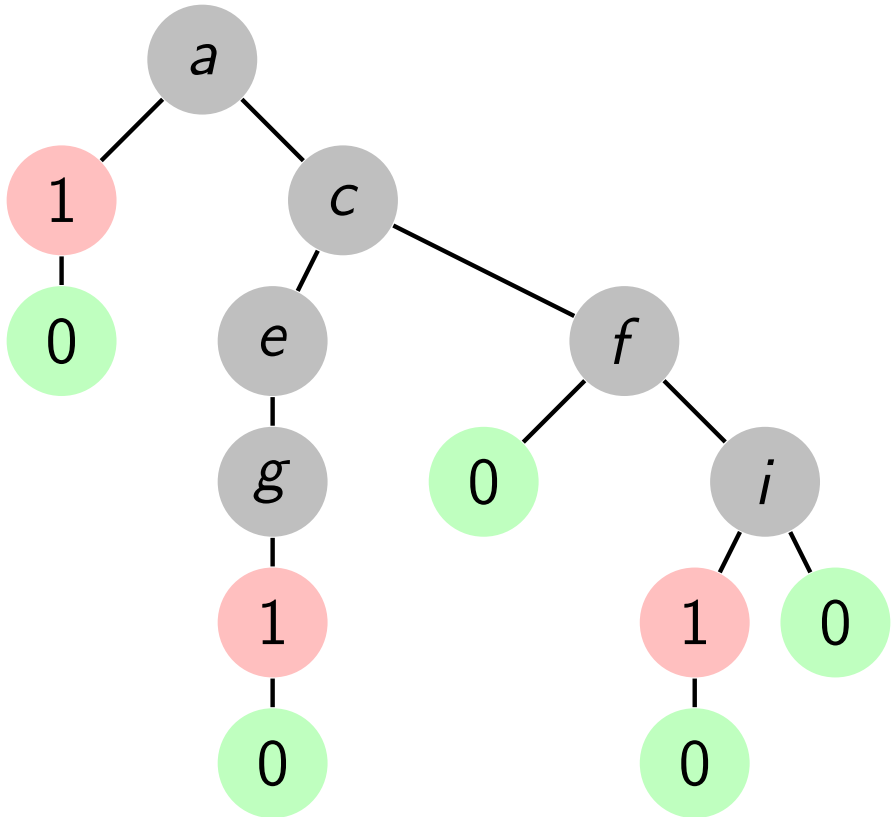

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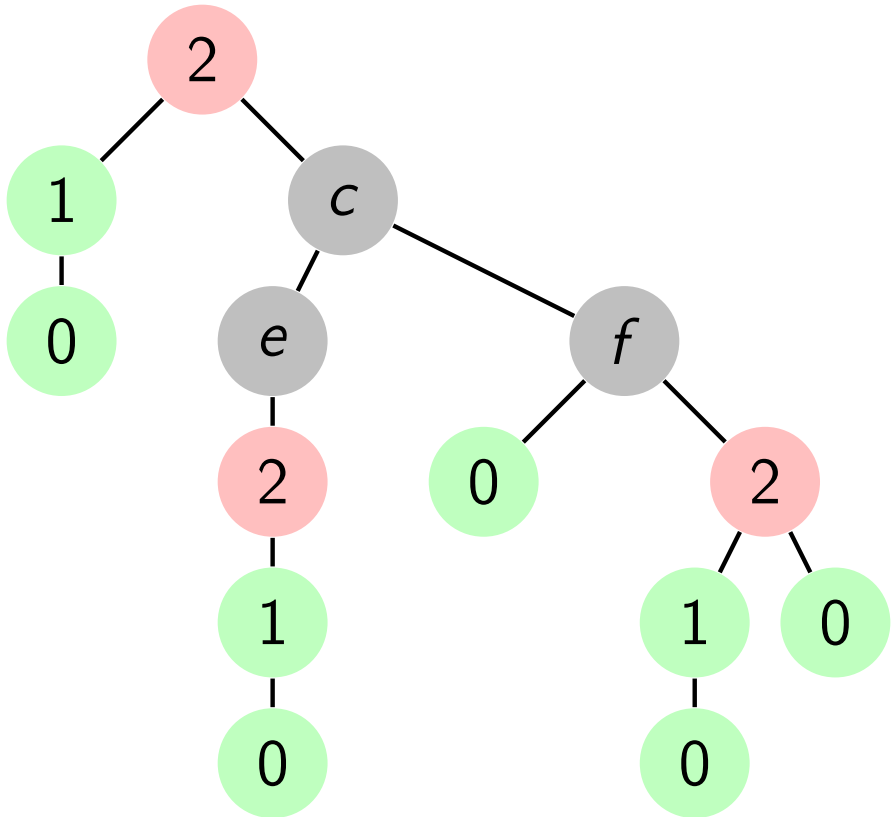

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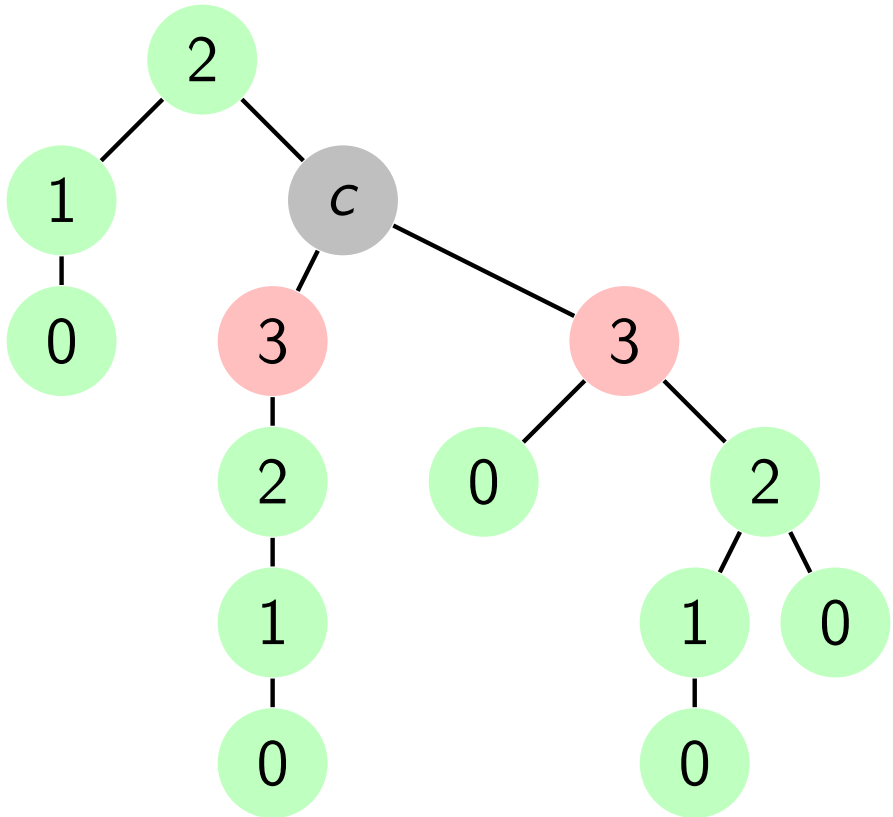

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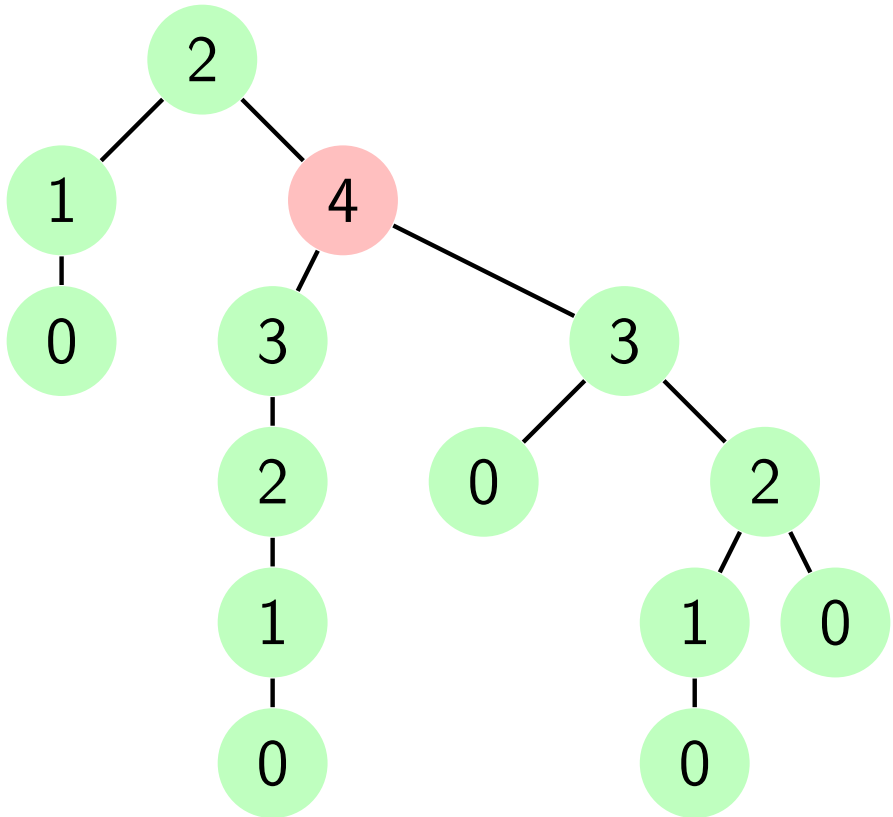

Award Ceremony