

BAPC 2021 Preliminaries

Solutions presentation

October 9, 2021

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Problem Author: Ragnar Groot Koerkamp

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- Naive solution: For each pair try whether x_i divides x_j . $\mathcal{O}(n^2)$ is too slow.
- Early break: stop as soon as you find a good pair. $\mathcal{O}(a/\ln(a)) \approx \mathcal{O}(10^8)$ expected steps is likely still too slow on the worst of the 100 test cases.

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- Greedy solution: Sort the input before doing the brute force with early break.
- Single pass solution: Keep the index of the smallest number seen so far, and check whether it divides the current number.
- Analysis:

The expected value of the smallest integer is $s \approx a/n = 4000$, so likely below 8000.

The probability that none of the $n = 5 \cdot 10^5$ integers is a multiple of $s \leq 8000$ is less than 10^{-27} .

If s does not work, we just try the next smallest integer. (But the probability of needing this is 10^{-5} , so only trying the smallest one is sufficient.)

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- Bonus solution: Use the birthday paradox.
The probability that all numbers in the list are distinct is only $7 \cdot 10^{-28}$, so we can just find and print the indices of two equal numbers.

B: Buffered Buffet

Problem Author: Boas Kluiving

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- **Solution:** Order guests by their required distance, then you can sum up their distances to get the table circumference.
 - The guest requiring the smallest distance is always satisfied on both sides, so this guest should not be counted.
 - The guest requiring the largest distance, requires this distance on both sides, so this guest should be counted twice.
 - All other guests are automatically satisfied on the side where somebody with lesser requirements is sitting, so they only need to be counted once.

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 - All other guests are automatically satisfied on the side where somebody with lesser requirements is sitting, so they only need to be counted once.
- **Final answer:** $\sum_i d_i + \max_i(d_i) - \min_i(d_i)$

C: Candy Contribution

Problem Author: Ruben Brokkelkamp

- **Problem:** Given a graph, nodes s and t , a number of candies c and for each edge e an integer p_e denoting what percentage of the candies you are carrying you have to pay to use the edge (rounded up).

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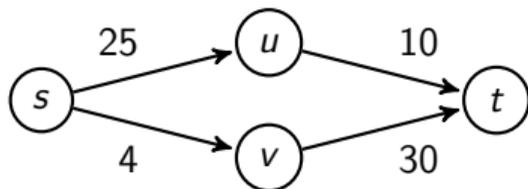
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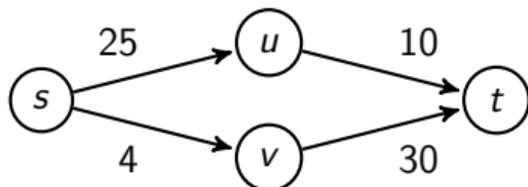
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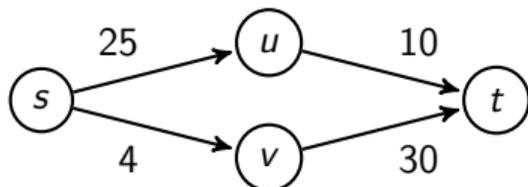


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- So, cannot do a 'normal' additive dijkstra with tax percentages to find best path.
- Solution: Tweak dijkstra a bit. Instead of initializing every node to ∞ and lowering it everytime you find a shorter path. Initialize everything to 0 and raise it when you find a path where you hold on to more candies.

D: Dickensian Dictionary

Problem Author: Mees de Vries

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- **Solution:** Check for every letter whether it is typeable with left or right
- Check if the resulting list is alternating
 - Note that you can start with either left or right

E: Entering Enemy Encampment

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- **Idea:** Use DP to find the score difference in remainder of the game, for every game state, assuming optimal play.
- **Issue:** $\mathcal{O}(3^n/\sqrt{n})$ game states, too many!

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- Vertices are either “claimed” or “unclaimed”, so only $\mathcal{O}(2^n)$ game states.
- Using a subset DP: $\mathcal{O}(2^n \cdot n^2)$ time needed.

F: Fridge Distraction

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- **Optimization:** You don't need to maintain a list if you do some math

G: Git mv

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- **Problem:** Given a file movement $s_1/s_2/\dots/s_n \rightarrow t_1/t_2/\dots/t_m$ find the shortest move description, assuming that the s_i are distinct and the t_j are distinct.

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- **Solution:** Greedy, i.e. find smallest i such that $s_i \neq t_i$ and smallest j s.t. $s_{n-j} \neq t_{m-j}$. Output:

$$s_1/s_2/\dots/s_{i-1}/\{s_i/\dots/s_{n-j}\} \implies t_i/\dots/t_{m-j}/s_{n-j+1}/\dots/s_n.$$

H: Histogram

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- **Solution:** First count the size for each bin, then print the histogram.
 - Make sure to calculate the height of the histogram beforehand

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- Sort the days by ice thickness [$\mathcal{O}(n \log(n))$].
- For each person binary search how many days have the required thickness [$\mathcal{O}(k \log(n))$].
- Alternative: store the number of days for each ice-thickness $\leq 10^6$, and accumulate once [$\mathcal{O}(k + n)$].

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- $\mathcal{O}(n^2 \cdot w)$ solution:
 - For each prefix of moles, compute all possible weights of a subset in $\mathcal{O}(n \cdot s)$.
 - For each suffix of moles, compute all possible weights of a subset in $\mathcal{O}(n \cdot s)$.
 - Mole i can be left out if it is possible to make a subset of size l with the moles before i , and a subset of size $(s - w_i)/2 - l$ of the moles after i , for some l .

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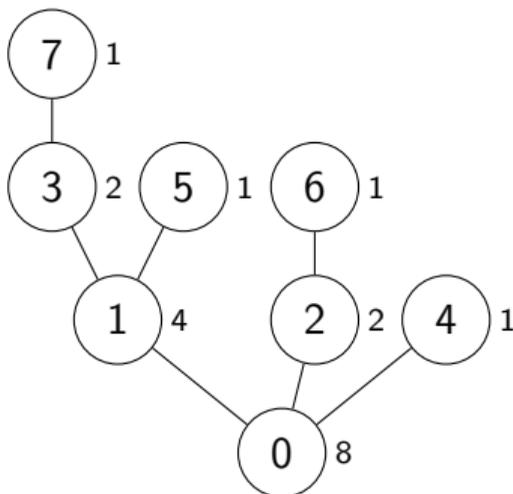
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- Solution: when removing vertex v with age i , return:

$$2^i - removed[v] \pmod{10^9 + 7}$$

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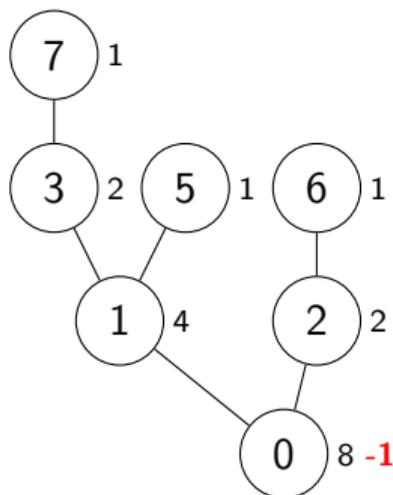
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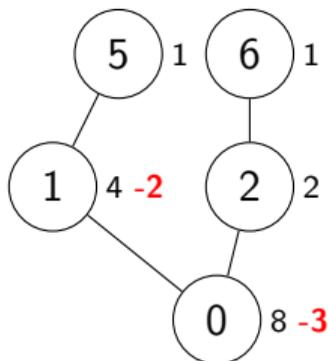
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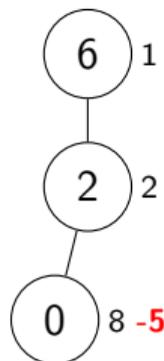
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Some stats

- 342 commits (last year: 527)
- 437 secret testcases (last year: 360)
- 175 jury solutions (last year: 221)
- The minimum number of lines the jury needed to solve all problems is

$$2 + 2 + 10 + 2 + 20 + 3 + 4 + 4 + 9 + 16 + 10 = 82$$

On average 7.5 lines per problem, down from 13.9 last year

Thanks to the Proofreaders!

Abe Wits

Nicky Gerritsen

Jaap Eldering

Mark van Helvoort

Kevin Verbeek

The Jury

Boas Kluiving

Erik Baalhuis

Freek Henstra

Harry Smit

Joey Haas

Jorke de Vlas

Ludo Pulles

Maarten Sijm

Mees de Vries

Ragnar Groot Koerkamp

Reinier Schmiermann

Robin Lee

Ruben Brokkelkamp

Timon Knigge

Wessel van Woerden