BAPC 2014 Solutions

Growling Gears

Jury Jeopardy

Button Bashing

Dropping Directions

Interesting Integers

Floating Formation

Excellent Engineers

Avoiding the Apocalypse

Citadel Construction

Key to Knowledge

Highway Hassle

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G - Growling Gears

Solution

- The maximum of $T(R) = -aR^2 + bR + c$ is at $R = \frac{b}{2a}$.
- The corresponding torque is $T = \frac{b^2}{4a} + c$.
- Find the gear for which this is maximal.

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J - Jury Jeopardy

Solution

- Start with a maze consisting of walls only.
- Track the robot's movements, add empty spaces wherever it goes.
- Find the limits of the maze and print it.
- Don't forget to print the number of test cases.
- ***Diff your output with example.out!***

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B - Button Bashing

Solution

- Construct a graph with a node for every amount of time.
- For each node, add edges corresponding to every button.
- Do a breadth-first search to find the shortest path to every node.

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D - Dropping Directions

Solution

- The road network consists entirely of loops.
- For every loop that doesn't contain the goal, at least one signpost is needed to get people off of it.
- For every loop that doesn't contain the goal, one signpost is sufficient.
- The answer is the number of loops that do not contain the goal.

I - Interesting Integers (1/3)

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Solution

$$G_1 = a$$
$$G_2 = b$$
$$G_3 = a + b$$
$$G_4 = a + 2b$$
$$G_5 = 2a + 3b$$

$$G_k = aF_{k-2} + bF_{k-1}$$

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I - Interesting Integers (2/3)

Problem description

Solve $n = aF_{k-2} + bF_{k-1}$

Method 1

- Try all b in increasing order.
- For each b, try all sufficiently small k, solve for a.
- b is at most $O(\sqrt{n})$.
- Complexity: $O(\sqrt{n} \log n)$

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I - Interesting Integers (3/3)

Problem description

Solve $n = aF_{k-2} + bF_{k-1}$

Method 2

$n = aF_{k-2} + bF_{k-1} = (b-a)F_{k-1} + aF_k = cF_{k-1} + aF_k \quad (c \geq 0)$

Try all k in decreasing order

$n = cF_{k-1} \pmod{F_k} \Rightarrow c = nF_{k-1}^{-1} \pmod{F_k} = nF_{k-1-k\%2} \pmod{F_k}$

Minimize c: $c = nF_{k-1-k\%2}\%F_k$

If $cF_{k-1} < n$ then:

$a = \frac{n-cF_{k-1}}{F_k}$ $b = a + c$

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F - Floating Formation

High-level problem

- First divide graph into 2-core and attached trees
- Then 'pin' leaves of trees, extending 2-core

Easiest solution

- Find attached trees through repeated peeling ($O(n)$)
- Find longest path down for every node ($O(n)$)
- Discount paths already on a path from a parent ($O(n)$)
- Take K longest paths ($O(n \log n)$, can do $O(n)$)

Can do several steps at the same time

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E - Excellent Engineers (1/2)

Problem description

Compute the 3-dimensional Pareto front.

Trivial $O(n^2)$ algorithm: TIMELIMIT.

$O(n \log n)$ time Solution

- Sort the engineers by skill 3, process them from good to bad.
- Maintain the 2-dimensional Pareto front of the engineers processed so far, i.e. the set of engineers for whom there has not been an engineer so far that is better in both skill 1 and 2.

E - Excellent Engineers (2/2)

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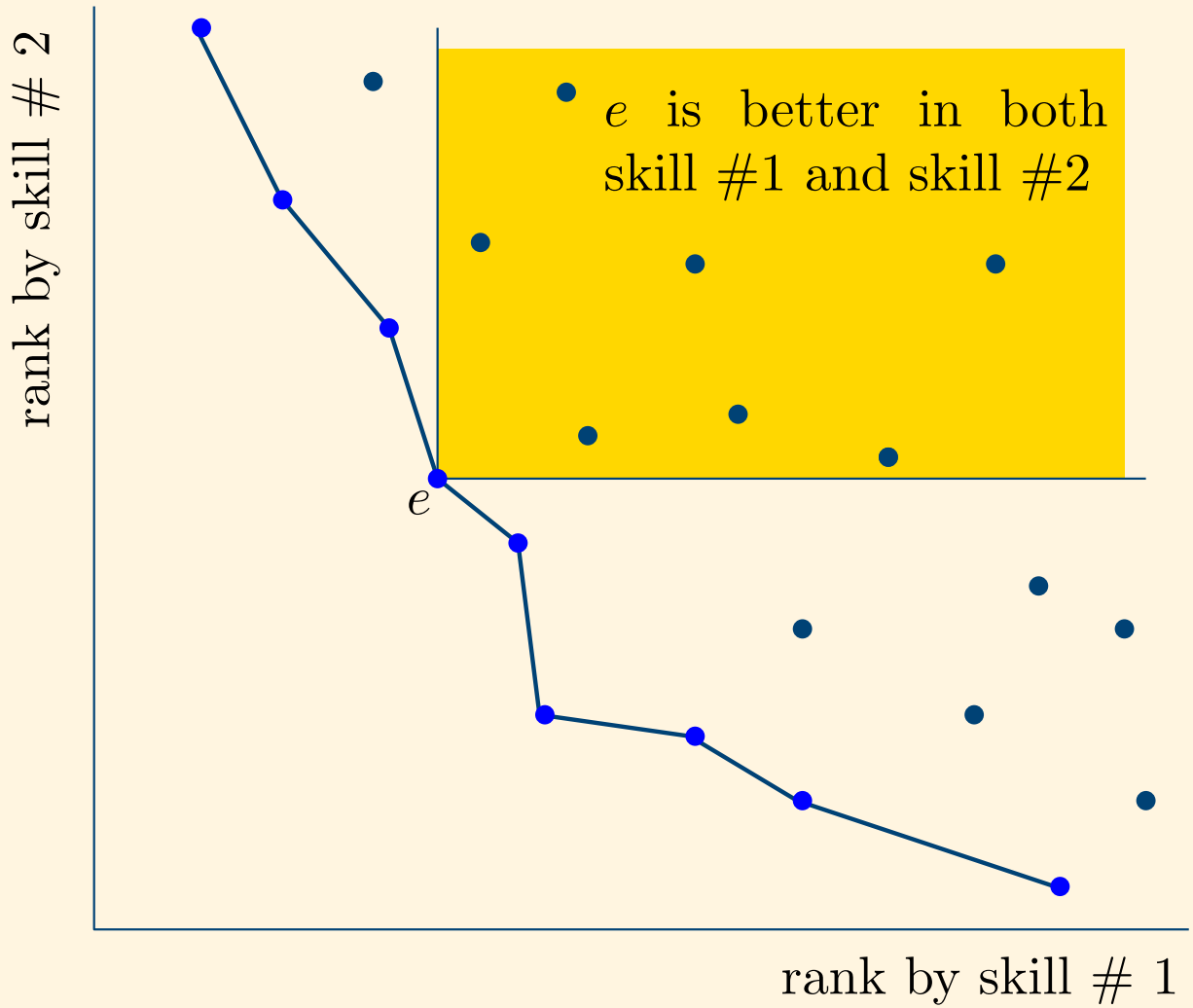
$O(n \log n)$ time Solution



E - Excellent Engineers (2/2)

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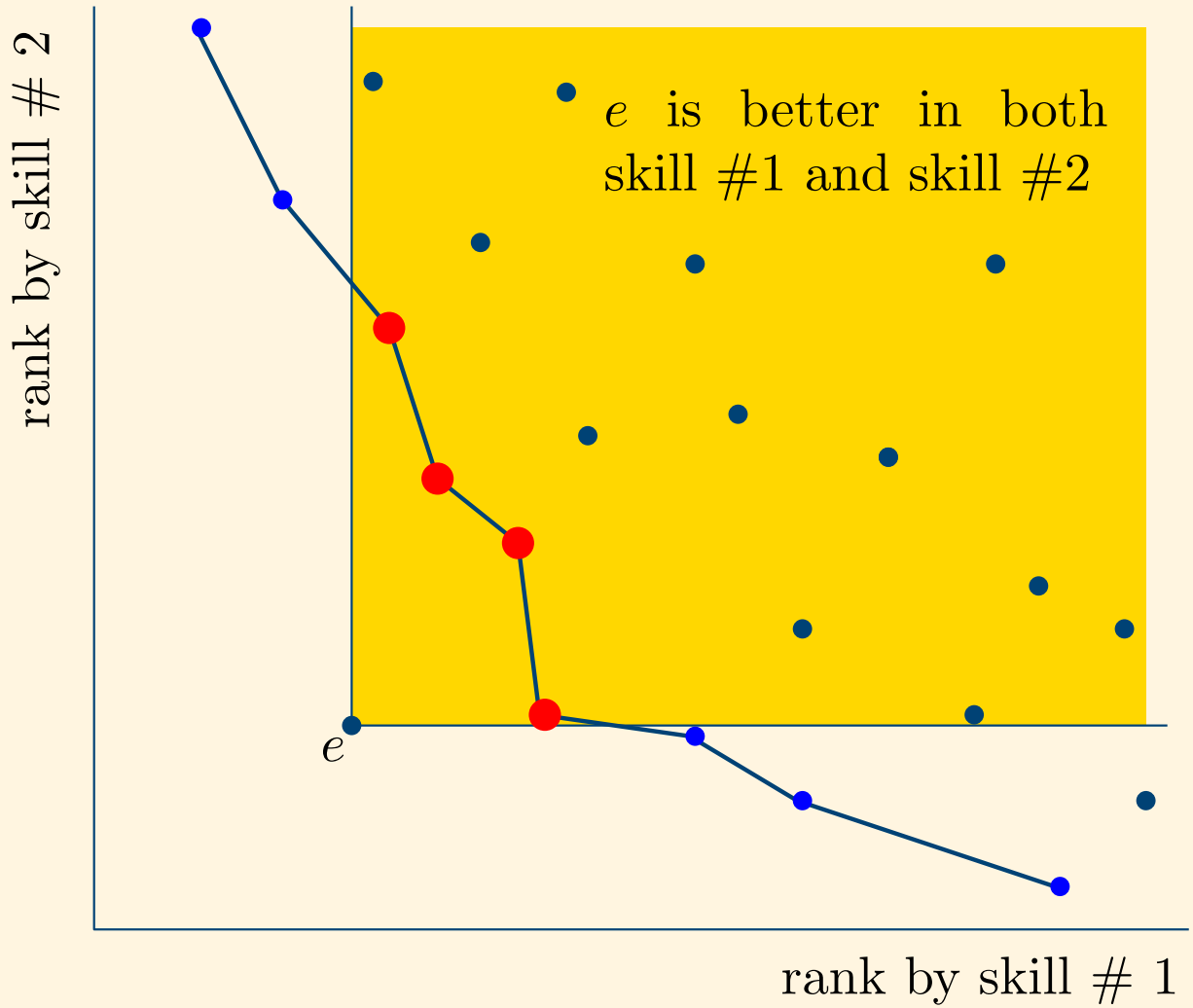
$O(n \log n)$ time Solution



E - Excellent Engineers (2/2)

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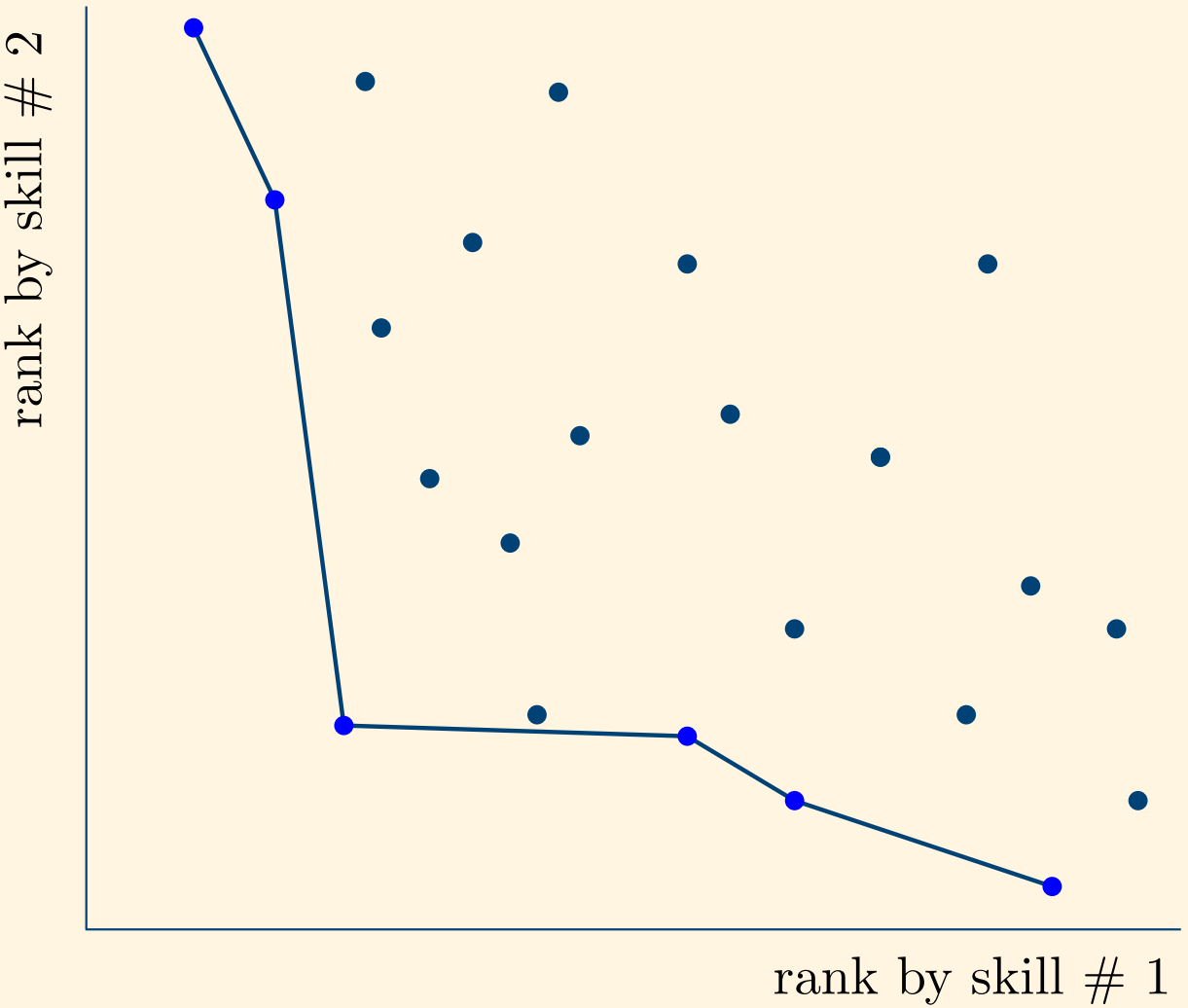
$O(n \log n)$ time Solution



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A - Avoiding the Apocalypse (1/2)

Solution

- Make a graph with nodes for every combination of location and timestep, a source and a sink.
- Add edges representing traversing a road at a certain timestep, with the corresponding capacity.
- Add edges representing waiting at a location for one timestep, with maximum capacity.

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A - Avoiding the Apocalypse (2/2)

Solution

- Add edges from each medical facility to the sink, with maximum capacity.
- Add an edge from the source to the start location at time zero, with capacity g.
- Determine the *maximum flow* from the source to the sink.
- Complexity: $O(\#edges \times |flow|) = O(rsp)$

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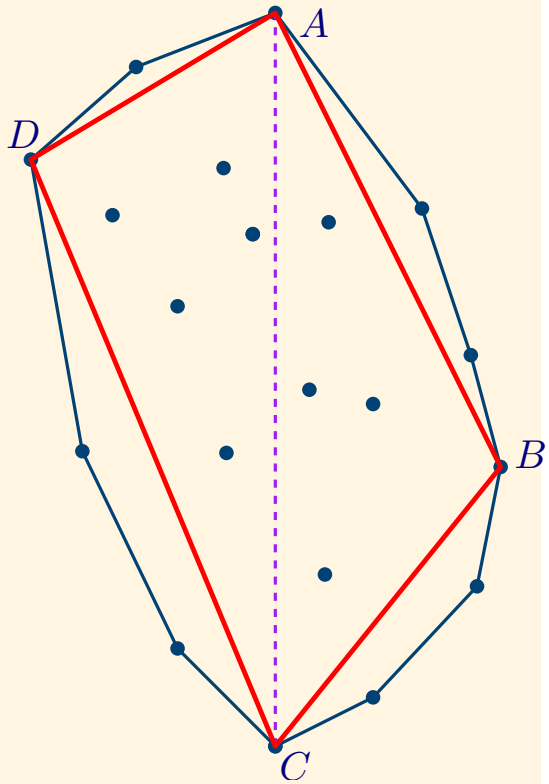
C - Citadel Construction

Problem description

Find the largest quadrangle.

Solution

- Find the convex hull.
- $\forall$ pairs of points (A, C) on the Convex Hull, find the two points that are furthest away from the line AC.
- The hull is convex, so this can be done in $O(\log n)$ time using ternary search.
- Complexity: $O(n^2 \log n)$ (but $O(n^2)$ and even $O(n \log n)$ are possible).



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K - Key to Knowledge (1/2)

Too slow

- Try all answers: $O(2^m)$
- With pruning: $O(\binom{m}{m/2})$

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K - Key to Knowledge (2/2)

Solution

- Split the questions into two (roughly) equal sets.
- For each set, try all answers and compute the number of correct answers for each student.
- Find pairs of answers that give the required total scores using the 2-sum algorithm:
  - Sort the answers in each set based on the student scores.
  - Go through one list from top to bottom and the other from bottom to top simultaneously.
- Complexity: $O(m^2 2^{m/2})$

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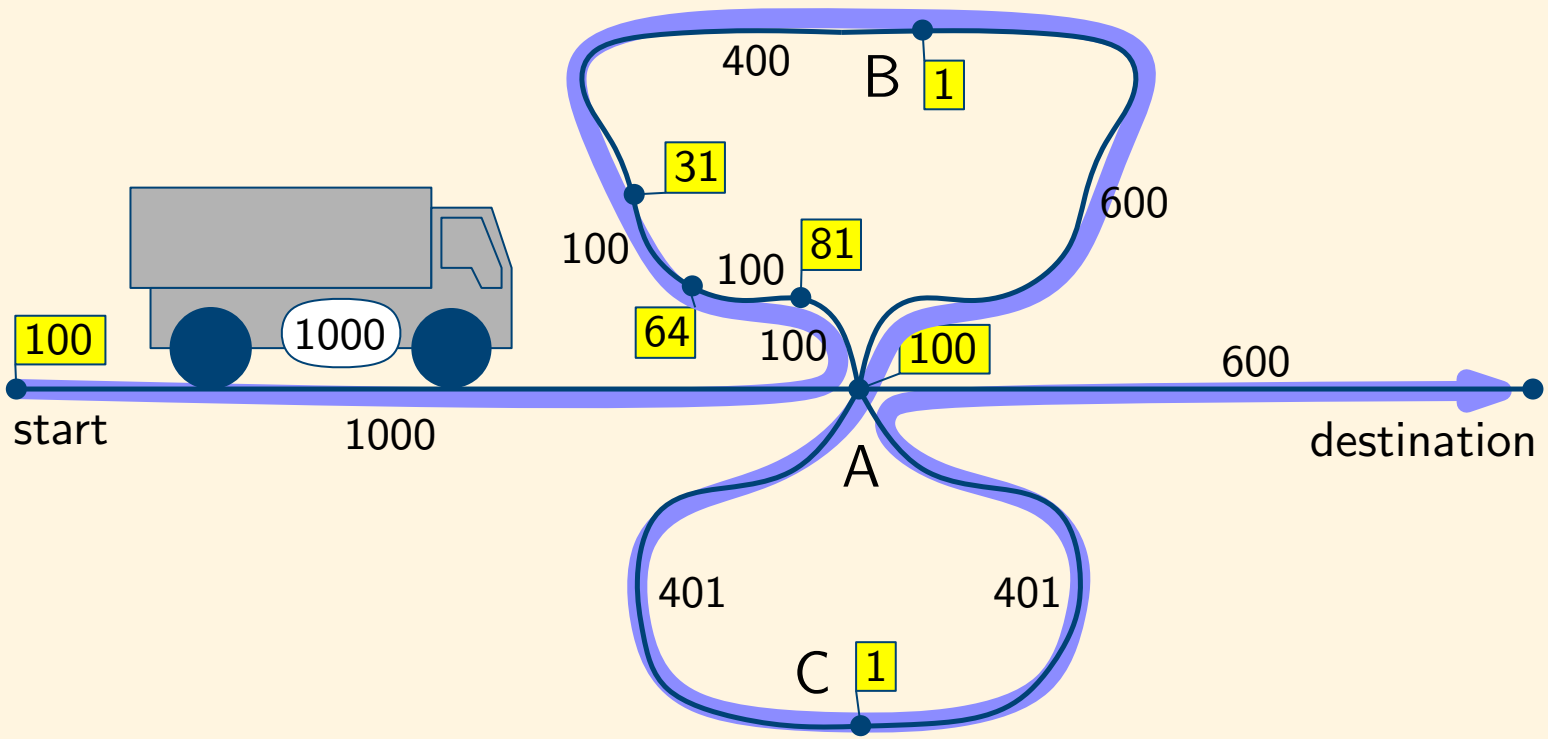
H - Highway Hassle (1/2)

Problem description

Find cheapest route + fuelling plan from start to destination.

May require ≥ 3 stops at the same petrol station



H - Highway Hassle (2/2)

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1. compute all distances between petrol stations (destination = free petrol station) in $O(s\, m \log n)$ time

   optimal: from expensive to cheap, take just enough petrol;
   from cheap to expensive, take full tank.

   $\rightarrow$ for each link you know tank contents at arrival/deparature.

2. search least-cost path in graph with node for each possible combination (station, amount of petrol at arrival/departure)