Segment tree contest
Problem analysis

Gleb Evstropov
glebshp@yandex.ru

November 14, 2015

1 Coding

A single operation of a cyclic shift can be reverted by performing another operation of a cyclic shift, i.e. $Shift(Shift(s, k), n-k) = s$. Cyclic shifts may be processed for $O(\log n)$ time if an array is stored in implicit-key Cartesian tree.

2 DFT for Dummies

One may notice that applying DFT operation twice changes the sequence $a_0, a_1, \ldots, a_{n-1}$ to $a_0, a_{n-1}, \ldots, a_1$. To solve the problem one needs to implement implicit-key Cartesian tree with `revert` operation.

3 K-th Maximum

Simply implement the Cartesian tree and store the size of the subtree in every node. To answer the query traverse down from the root, going left or right, whether there are enough nodes in the left subtree.

Also, as the problem is offline (i.e. all queries are known at the beginning) we can replace Cartesian tree with the segment tree.

4 Luxury Burrow

Standard trick for all problems requiring to optimize some function is to use binary search. For some fixed value b we replace all elements $a_{ij} \geq b$ with ones and all elemenents $a_{ij} < b$ with zeroes. We pay $\log C$ multiplier in complexity to reduce the problem to the following: is there a rectangle with square $\geq k$ containing only ones?

There is a standard algorithm for finding the rectangle of the maximum area consisting of ones:

- Use scanline, one-dimensional problem sounds: given an array a_i maximize the value of $\min\limits_{j \in [l,r]}(a_j) \cdot (r - l + 1)$.
- For every i find the nearest element to the left and the nearest element to the right the are strictly lesser. This can be done using the stack-based algorithm, same as building Cartesian tree in a linear time.

5 Period

- Only prefixes whose length is a divisor of $|s|$ are candidates to be a period of the string;
- To check if the prefix of length k is a period we need to compare strings $s[1 \ldots n-k]$ and $s[k+1 \ldots n]$, where $n = |s|$;
- The above comparison can be done by computing polynomial hashes;
- As the string is the subject to change, we need to store the value of polynomial hash in nodes of a Cartesian tree;
- Current solution works in $O(q_? \cdot d(n) \cdot \log n + (q_+ + q_-) \cdot \log n)$;
- If a is a period and b is a period, than $gcd(a, b)$ is a period too;
- We do not need to try all divisors, just all the primes in the factorization of n. Total complexity is $O(q_? \cdot \log^2 n + (q_+ + q_-) \cdot \log n)$

6 Urns and Balls

Instead of directly processing all the balls, we will compute the function $\phi(x)$ — where will the ball finish, if it starts if the urn x.

$$\phi(x) = \varphi_m(\varphi_{m-1}(\ldots \varphi_1(x) \ldots))$$

If we process the move operations from the last to the first, on the i-th step we will have:

$$\phi_{m-i}(x) == \varphi_m(\varphi_{m-1}(\ldots \varphi_{m-i}(x) \ldots))$$

Applying a single move from $(s_i,\ s_i + l_i)$ to $(f_i,\ f_i + l_i)$ is equal to setting:

$$\phi_{m-i-1}(x) = \phi_{m-i}(x) \text{ for } x < s_i \text{ or } x > s_i + l_i$$
$$\phi_{m-i-1}(x) = \phi_{m-i}(x - s_i + f_i) \text{ for } s_i \leq x \leq s_i + l_i$$

Keep calm, and feel the power of implicit-key persistent Cartesian tree.