Long Contest Editorial

November 14, 2015

Moscow International Workshop ACM ICPC, MIPT, 2015

A. Automorphism

Given a tree, count the number of its automorphisms.

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Note that different profiles correspond to non-isomorphic trees, and vice versa.
Store a global list of different profiles, with the mapping from profiles to unique numbers. If the current profile was not met before, assign it a new number and remember the profile.

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If there are k isomorphic subtrees of some type, we multiply the answer by $k!$.
Together we mapping numbers and building profiles, this algorithm works in $O(n \log n)$.

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Definition

A vertex v is called a *centroid* of the tree if all its subtrees contain at most $n/2$ vertices.

Fact

A tree may only contain a unique centroid, or a pair of adjacent centroids.

Observation

Any automorphism maps a centroid to a centroid (possibly the same).

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Note that assigning unique numbers to non-isomorphic trees can be done in linear time (without hashing).

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In a rectangle there are several vertical and horizontal beams. Ball moves inside the rectangle, reflecting from its sides after collisions. For several values of initial position, velocity and time interval, count the number of different intersection moments with beams. We can shrink the ball and expand the beams equally. After that, we can assume that a point flies in the rectangle and collides with vertical and horizontal strips.

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The problem for one type of beams is fairly easy: for, say, vertical beams we can use reflection principle over vertical edges to obtain a periodical infinite sequence of strips. After that, counting reduces to range queries on the periodical array.
However, we counted double collisions twice, and now have to subtract them back.

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This reduces to counting the number of pairs of vertical and horizontal strips at offsets x and y such that $x - y = d$ or $x + y = d$ depending on direction of the diagonal.
These numbers can be counted all at once using FFT for fast polynomial multiplication.

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As the number of segments can be large, we should use faster summation methods. For example, count the DP "total number of double collisions starting at current segment and following 2^k segments forward" to sum along path of length l in $O(\log l)$ time.
It suffices to account for proper parts of start and finish segments.
This can be done using bitmasks in $\sim ((n+m)/32)$ time.

C. SmartDog

A dog and a cat are placed inside a labyrinth. On every turn cat moves to a random adjacent cell, and a dog either hears nothing or hears a distorted located where the cat is located. After that, dog moves so that to minimize expected shortest path to cat (to its knowledge). Given what dog hears, simulate dog's movements.

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- Suppose that on that current move dog heard nothing.
  Consider a cell (x, y).
  Then, consider all adjacent cells (let their number be d). For each adjacent cell (x', y'), add $p(x, y)/d$ to $p'(x', y')$; this corresponds to the uniform choosing of direction.

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  Let $L(x_i, y_i, t_i)$ be the event "the cat is at the position (x_i, y_i) at the moment t_i"
  Let $H(x_i, y_i, t_i)$ be the event "the dog heard noise at the position (x_i, y_i) at the moment t_i".

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For some cell (X, Y), $p'(X, Y)$ should be equal to $\mathrm{Prob}(L(X, Y, t) | H(x, y, t), H(x_1, y_1, t_1), \ldots)$, where (x_i, y_i, t_i) are all previous events "dog heard noise".

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Note that denominator is the same for all cells (X, Y), so we can omit it and normalize the distribution later.

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$\mathrm{Prob}(H(x,y,t)|L(X,Y,t))$ is by definition $Ne^{-((x-X)^2+(y-Y)^2)/2\sigma^2}$.

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$Ne^{-((x-X)^2+(y-Y)^2)/2\sigma^2}$.
To sum up, $p'(X,Y)$ is just $p(x,y)$ multiplied by probability of hearing noise at (x,y) if the cat is at (X,Y) (which makes sense).

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Finally, for each option of movement calculate expected shortest path to the cat and choose the best. Pay attention to tolerance of comparison between probabilities.

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Finally, for each option of movement calculate expected shortest path to the cat and choose the best. Pay attention to tolerance of comparison between probabilities.
Complexity is $O(knm + (nm)^3)$ (the latter summand for calculating all shortest paths).

D. Expression

Given an arithmetic expression, erase as much bracket pairs as possible so that the expression keeps its original meaning.
More specifically, we can assume that multiplication/division has higher priority than addition/subtraction, and addition and multiplication are associative. Also, subtraction can be combined with addition if it is a second operation, and similarly division can be combined with miltiplication.
No more assumptions can be made about the operations.

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We will use following notation (context-free grammar):

- term $:=$ $a..z|(\text{expression})$
- product $:=$ $\text{term}|\text{term} * \text{product}|\text{term}/\text{product}$
- expression $:=$ $\text{product}|\text{product} + \text{expression}|\text{product} - \text{expression}$

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- If the expression inside of brackets is a letter, then the brackets can be removed indefinitely.
- If the expression inside of brackets is a product (according to the described context-free grammar), the brackets can be removed iff there is no division (/) immediately before the term t.
- Otherwise, the brackets can be removed if the term t is not a proper part of the product (that is, does not take part in the multiplication or division immediately), and also there is no $-$ immediately before the term t.

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These can be checked easily along with parsing the expression in linear time.

E. Paint

Paint the edges of a graph into white, blue and red in such a way that every white edge is adjacent to a red edge. Minimize total cost if a white edge costs 0, a blue edge costs 1, and a red edge costs 2.

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For each of the 2^n subsets we will count the minimal number of red edges needed to cover this set. This can be done with a classical subset DP, with transitions "add a single edge".

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Having computed this, for each subset count the number of blue edges explicitly.
In total, we obtain a $O(m2^n)$ solution.

F. Parliament

Given a graph with “friend” and “enemy” edges, find the maximal size of a subset S such that friends of elements of S are also in S, and no enemy of an element of S is in S. Also, count the number of different maximal subsets.

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The number of “friend” edges is at least $\frac{n(n-1)}{3}$.

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For $n \leqslant 250$, this number does not exceed 46.

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After compressing components, the problem becomes basically equal to the maximal independent set problem, but with weights. This problem can solved in $O(n2^{n/2})$ on a graph with n vertices with a “meet-in-the-middle” approach and fairly standard subset DP.

The total complexity is roughly $O(n2^{(1-\sqrt{\frac{2}{3}})/2})$.

G. Sequences

We are given an array of n numbers. Consider all p-element decreasing sequences and order them lexicographically by sequences of indices. Find k-th lexicographically p-element decreasing sequence for several values of k.

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- Determine the first element by moving from left to right and comparing k with $A_{p,i}$. If current $A_{p,i} > k$, then i is the index of the first element, otherwise subtract $A_{p,i}$ from k and continue.
- Determine all other elements similarly, but skip elements that are not less than the previous chosen element.

G. Sequences

First, we will count $A_{q,i}$ the number of q-element decreasing sequences starting at the index i for all indices, and all values of q from 1 to p.
This is a standard application of Fenwick tree (BIT) or segment tree.
Given this information, we can find k-th (0-based, for convenience) decreasing sequence as follows:

- Determine the first element by moving from left to right and comparing k with $A_{p,i}$. If current $A_{p,i} > k$, then i is the index of the first element, otherwise subtract $A_{p,i}$ from k and continue.
- Determine all other elements similarly, but skip elements that are not less than the previous chosen element.

This allows to answer one query in $O(n)$ time, which is too much to process all queries.

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- Sort all events "find the next element of j-th sequence with previous element a_i" and "add element a_i into consideration" so that for each query with element a_i only larger elements are added into BIT.
- Process queries with binary search similar to finding the first element (see above).

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This algorithm will answer all queries in $O(np \log n + qp \log^2 n)$. This can be further optimized to $O((n+q)p \log n)$ using tree descent instead of binary search with queries (note that both segment tree and BIT approach can be optimized this way).

H. Squares

Given a set of points in the plane, count different squares with vertices in given points and sides aligned with coordinate axes.

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- If there are few (say, at most k) points with $x = x_l$, we will try all possible pairs of them as left-top and left-bottom vertices. If these points are (x_l, y_l) and (x_l, y_r), then the side length of the square is equal to $y_r - y_l$, and other two poitns should be $(x_l + y_r - y_l, y_l)$ and $(x_l + y_r - y_l, y_r)$.

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  We can check if these two points exist in $O(\log n)$ with binary search or `std::set`-like data structure.
  Complexity for each particular x is $O(k^2 \log n)$, for a total complexity of $O(nk \log n)$, since sum of all k's doesn't exceed n.

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- If there are more than k points, we will choose a different strategy. If we choose the right-top point (x_r, y_r) along with x_l, all other points are determined unambigiously. Check if they exist in a similar matter.

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  The complexity is $O(\frac{n}{k} n \log n)$, since there are at most $\frac{n}{k}$ x_l's with more than k points.

Choosing $k \sim \sqrt{n}$, we obtain the solution with total complexity $O(n\sqrt{n} \log n)$.

I. Tables

We are given k permutations $p_{i,j}$ of n elements. Construct a permutation q on n elements such that

$$\sum_{i=1}^{k}\sum_{j=1}^{n}\min(|p'_{i,j} - q'_j|, 8)$$

is minimized, where p'_i and q' are inverse permutations of p'_i and q'.

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Thus we obtain a graph with $O(n)$ vertices and edges. Still, majority of min-cost max-flow algorithms will time out.
However, this problem can be reduced to regular max-flow with clever network construction.
As the network is bipartite, we can use Dinic algorithm to achieve $O(nk\sqrt{n})$ complexity (however, the constant factor is large as the number of edges is roughly $16kn$).

J. Triangle

Count the number of integer points with non-negative coordinates such that $ax + by \leqslant c$ for integer a, b, c.

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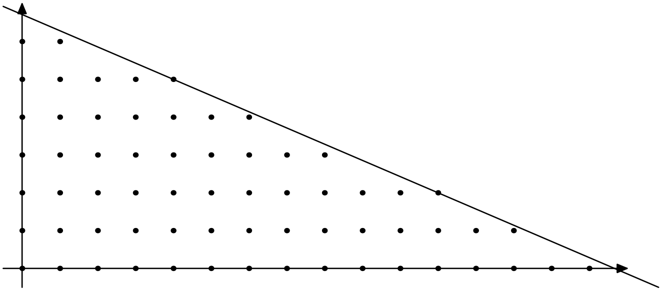
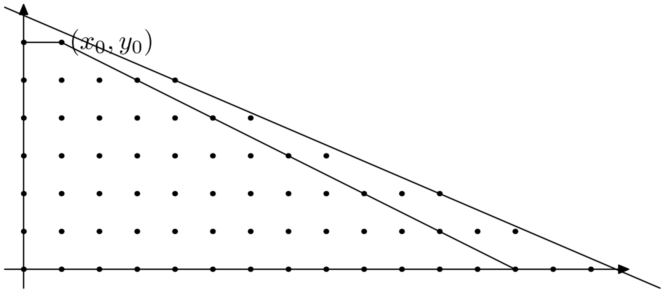


Consider region $ax + by \leqslant c$, $x, y \geqslant 0$.

J. Triangle



Denote $y_0 = \lfloor \frac{C}{B} \rfloor$, $x_0 = \lfloor \frac{C - By_0}{A} \rfloor$. Point (x_0, y_0) is the rightmost of the top points inside the region.

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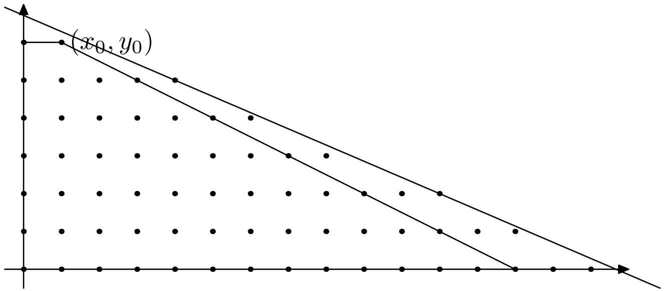

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Let $k = \lfloor \frac{B}{A} \rfloor$. The orthogonal trapezoid with top-right point (x_0, y_0) and right side along the vector $(k, -1)$ is inside the region.

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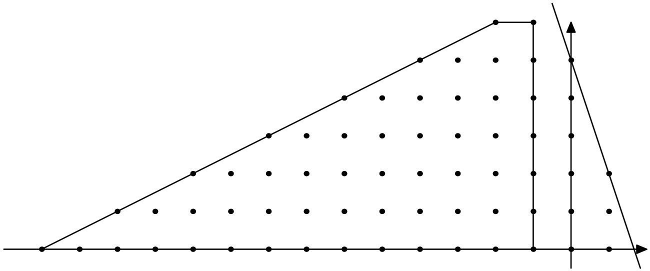

Apply coordinate tranformation $x \to x - k \cdot (y_0 - y) - x_0 - 1$. This transformation takes the points inside the trapezoid to points with negative x, the number of such points can be counted explicitly.

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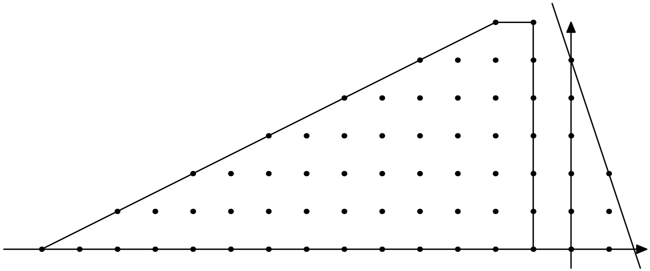

Apply coordinate tranformation $x \to x - k \cdot (y_0 - y) - x_0 - 1$. This transformation takes the points inside the trapezoid to points with negative x, the number of such points can be counted explicitly. The rest is a region $a'x + b'y \leqslant c'$, where $b' = b$, $a' = b - ka$. Count the number of points in this region recursively.

J. Triangle

Since we're basically applying Euclid's algorithm to A and B, the number of iterations, and thus the total complexity will be $O(\log(a+b))$.